P. 5351. Vajon miért nem szabad a lézerfénybe belenézni?

Az ember szemlencséje a fényt igen kicsi felületre, jellemzően néhány μm-es tartományra képes fókuszálni. A legérzékenyebb sejtek a retinában vannak, itt a „csap” és „pálcika” nevű idegsejtek mérete a μm-es tartományba esik. A mindennapi életben használt lézerek teljesítménye 0,1 mW és 100 mW között van.

Számítsuk ki, hogy a legkisebb, tehát 0,1 mW teljesítményű lézer fénye 80%-os fényelnyelés mellett mennyi idő alatt melegít fel egy sejtet a károsodást okozó $50\,^\circ$C-ra, és mennyi idő alatt a biztos roncsolást okozó $100\,^\circ$C-ra. Az egyszerűség kedvéért tekintsünk egy idegsejtet 5 μm átmérőjű és 7 μm mélységű hengernek, amelynek sűrűségét és fajhőjét a vízével vehetjük egyenlőnek. A szem hőmérsékletét vegyük 36 °C-nak, és egyéb hatásokkal (elmozdulások, hővezetés stb.) most ne törődjünk. A kapott időt vessük össze az emberi szem kb. 0,2 másodperces reakcióidejével!

(*4 pont*) Közli: *Vass László,* Budapest

P. 5352. Egy R ellenállású, A keresztmetszetű, zárt körvezetőt B indukcióvektorú mágneses térben szeretnénk forgatni a síkjában lévő szimmetriatengelye körül állandó ω szögsebességgel. Mekkora átlagteljesítménnyel tudjuk ezt megtenni?

(*4 pont*) Közli: *Szász Krisztián,* Budapest

P. 5353. Mi az oka annak, hogy a kibányászott uránérc aktivitása jelentősen nagyobb, mint a belőle készülő uránsóé?

(*4 pont*) Közli: *Simon Péter,* Pécs

P. 5354. Motoros játékvonat halad R sugarú, kör alakú pályán, állandó nagyságú v sebességgel. A kör középpontjától $d<R$ távolságra egy állandó, f_0 frekvenciájú hangot kibocsátó, pontszerű hangforrás helyezkedik el. A vonatra egy mikrofont rögzítünk. Milyen határok között változik a mikrofon által észlelt hang frekvenciája? (A hang sebessége c.)

(*6 pont*) Közli: *Vigh Máté,* Biatorbágy

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Problems in Mathematics

New exercises for practice – competition K (see page 412): **K. 699.** We have six discs. Each disc has a letter on one side (A, B, C, D, E, F), and a number on the other side (1, 2, 3, 4, 5, 6, in some order). The discs are placed on the table with their letter side up. Given that the sum of the numbers on the discs marked A, B and C is 14, and the sum of the numbers on discs A, D and E is 12, what is the minimum number of discs to be turned over in order to know which number is on which disc? **K. 700.** We have ten cards numbered 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. The cards are placed on the table in a row, and their numbers are added up in sets of three: first the cards in positions 1, 2, 3; then those in positions 2, 3, 4; followed by positions 3, 4, 5; and so on; finally adding up the numbers on the cards in positions 8, 9, 10. The sums obtained in this way are 14, 18, 24, 23, 24, 21, 16, 12, in this order. What is the sum of the numbers on the cars in the first and last positions? **K. 701.** A flea is sitting on the 0 mark of the number line, ready to jump. With each jump of the flea it moves 3 or 5 units either to the left or to the right. The flea needs to visit every integer from 1 to 20. Find a possible sequence of at most 22 jumps that will let the flea achieve that goal. **K/C. 702.** Five cards were drawn from a deck of 52 French cards. It turned out that none of them are face cards, and there is at least one card from each suit. The sum of the even denominations among them is equal to the sum of the odd denominations. The sum of the spades is 14, the sum of the red cards is 10, and the card with the lowest denomination is a heart. Which cards were drawn? **K/C. 703.** In a positive decimal fraction, the decimal point is shifted four places to the right. The resulting number is four times the reciprocal of the original number. What is the original number?

New exercises for practice – competition C (see page 413): **Exercises up to grade 10: K/C. 702.** See the text at Exercises **K. K/C. 703.** See the text at Exercises **K. Exercises for everyone: C. 1684.** Prove that there exists no pentagon in which all sides are equal in length and two angles are 60°. **C. 1685.** In a royal dynasty, there are eight brothers. The present king is the eldest brother. As a rule, a brother will come to the throne when he is the oldest of those alive. However, there is a curse on the dynasty: whenever each of three successive brothers comes to the throne, the following brother will die from despair. In how many different ways may the brothers rule? (Only the set of those coming to throne matters.) **C. 1686.** The hypotenuse of the right-angled triangle ABC is AB. The interior angle bisector f drawn from vertex A intersects side BC at point D. Prove that the geometric mean of line segments $AB-BD$ and $AC+CD$ equals the length of angle bisector $f=AD$. (Proposed by *N. Zagyva,* Baja) **Exercises upwards of grade 11: C. 1687.** I found three shopping lists in a shopping bag. The first list included 23 buns, 13 apples and 15 eggs, the second list had 9 buns, 3 apples and 28 eggs, and the third one had 25 buns, 18 apples and 11 eggs. The amount paid for these items on list one was 2021 forints (HUF, Hungarian currency), and the items on lists two and three cost 2031 and 2041 forints, but I cannot remember which sum belongs to which list. Each of the three kinds of products costs a whole number of forints a piece. What is the piece price of each item? (Proposed by *M. E. Gáspár,* Budapest) **C. 1688.** The single mode of a set of data is 2, the median is 3, the mean is 4, and the range is 5. How many elements may the data set have?

New exercises – competition B (see page 414): **B. 5190.** In a table of n rows and k columns, there is -1 written in each field. In each move, one row and one column is selected. Each number in the row is changed to the opposite, and then each number in the column is changed to the opposite. For what values of n and k is it possible to achieve a value of $+1$ in every field of the whole table? (*3 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 5191.** We have a wooden set square, but the hypotenuse has been chewed by a rabbit. With the set square we can join points lying close enough, we can extend straight line segments, and we can draw a perpendicular to any line at any point. Can we construct the centre of a given circle however large its size? (*4 points*) (Proposed by *M. E. Gáspár,* Budapest) **B. 5192.** Eight boys decided to play a game of football, four against four, on each of the first seven days of the autumn break. Is it possible to organize the teams so that any set of three boys would play in the same team at least once? (*5 points*) (Based on the idea of *M. E. Gáspár,* Budapest) **B. 5193.** In an acute-angled triangle ABC, $\angle BCA = 45^\circ$, the feet of the altitudes on sides BC, CA, AB are D, E, F, respectively, and the orthocentre is M. Point F divides line segment AB in a ratio $AF : FB = 2 : 3$. G is the point on side AC for which $CG = BM$. Show that the centroid of triangle ABG is M. (*4 points*) **B. 5194.** In a triangle ABC, $\angle ABC = 2\angle CAB$. Side AB touches the inscribed circle at point E, and intersects the angle bisector drawn from C at point F. Prove that $AF = 2BE$. (*4 points*) **B. 5195.** Prove that the inequality $x^p \cdot y^{1-p} < x + y$ holds for every pair of positive real numbers (x, y), and all real numbers $0 < p < 1$. (*3 points*) **B. 5196.** Let $p(x) = 2x + 1$. A is a subset of set $S = \{1, 2, \dots, 2021\}$ such that it contains at most one of the numbers n, $p(n)$, $p\big(p(n)\big)$ for every n, but this condition will not hold anymore if any extra element of S is added to A. What may be the number of elements in the set A? (*6 points*) **B. 5197.** Let $\mathbb{N}$ denote the set of non-negative integers, and let k be a given positive integer. Is there a monotonically increasing function $f : \mathbb{N} \to \mathbb{N}$ such that $f\big(f(x)\big) = f(x) + x + k$ for all $x \in \mathbb{N}$? (*6 points*)

New problems – competition A (see page 415): **A. 806.** Four distinct lines are given in the plane, which are not concurrent and no three of which are parallel. Prove that it is possible to find four points in the plane, A, B, C and D with the following properties: (*i*) A, B, C and D are collinear in this order; (*ii*) $AB = BC = CD$; (*iii*) with an appropriate order of the four given lines A is on the first, B is on the second, C is on the third and D is on the fourth line. (Proposed by *Kada Williams,* Cambridge) **A. 807.** Let $n \geqslant 2$ be a given integer. Let G be a finite simple graph with the property that each of its edges is contained in at most n circuits. Prove that the chromatic number of the graph is at most $n + 1$. (Proposed by *Ádám Schweitzer,* Budapest) **A. 808.** Find all triples of positive integers a, b and c such that a, b and c are pairwise relatively prime and $a^2 + 3b^2c^2 = 7^c$. (Proposed by *Nikolai Beluhov,* Bulgaria)

Problems in Physics

(see page 443)

M. 407. An EPS panel is a set of compressed small styrofoam balls. If the panel is broken or sawn, such small balls can easily fall out of it. Measure how many times the density of some of these pellets is greater than the density of the EPS panel.

G. 753. Two 5-metre long vehicles are travelling one after the other on a highway at a speed of 100 km/h. The distance between the cars is 30 m. Once, the car at the back starts overtaking. It accelerates uniformly until the two cars are next to each other. At this moment the speed of the accelerating car is 130 km/h, which remains constant for the rest of the motion. This car finishes the overtaking manoeuvre by positioning itself 30 m ahead of the other car moving at a constant speed. How long did the overtaking last? **G. 754.** Newspaper news on March 20, 2021: "The vast majority of space debris revolve around the Earth at low orbits, i.e. from an altitude of 800 km up to 2000 km, at a speed of 28 000 km/h." *a*) At what altitude can a piece of space debris orbit at a speed of 28 000 km/h? *b*) At what speed can a piece of space debris orbit at an altitude between 800 to 2000 kilometres? **G. 755.** An 80 kg action hero uses a parachute that sinks at a speed of 8 m/s when open. In one scene, he catches the heroine, who weighs 60 kg, in the air and then he opens the parachute. At what speed does the clinging pair reach the ground? From what height should they jump without parachute in order to reach the ground at the same speed? **G. 756.** The gauge pressure in the tyre of a car measured by a meter at a gas station is 1.2 bars. Assuming that neither the volume of the tyre nor