A Nutrition Adviser's Menu Planning for a Client Using a Linear Optimization Model

Lucie Schaynová

University of Ostrava, Faculty of Science, Department of Mathematics
30. dubna 22, 70103 Ostrava, Czech Republic
lucie.schaynova@osu.cz

Abstract: This paper presents a new linear optimization model which improves a nutritional adviser's work and prevents mistakes when preparing a diet plan for a client manually. The model takes the client's favourite or the adviser's recommended recipes into account, prevents unbalanced nutrition, respects the client's eating habits and habits of measuring when cooking, ensures recommendations for people from the Czech Republic and prevents wasting food items. The model also ensures that the client's daily recommended intake of nutrients is met, that certain nutrients are balanced in proportion when applicable, and that the energy intake is distributed during the whole day. The model involves linear constraints to ensure that two incompatible recipes are not used in the same meal and that a recipe is not used in an incompatible meal. A corresponding balanced feeding plan is produced for the client for several days. The solution will yield particular recipes for particular days with the exact amounts of the food items used. The final dietary plan for the client is optimal.

Keywords: linear programming; diet problem; nutrient requirement; menu planning; nutrition adviser

1 Introduction

The question of feeding people is a fundamental question for the entire planet: an estimated two-thirds of the world's population suffers from various degrees of nutrition deficiency (malnutrition[1]). This nutrition deficiency is caused by starvation, quantitative and qualitative insufficient nutrition as well as faulty and unbalanced nutrition. It is also linked to bad habits, such as overeating. People

[1] Malnutrition is a bad nutrition state of a client. It is caused by insufficient intake of basic nutrients (proteins, saccharides and fats) as well as vitamins, minerals and trace elements.

with gastrointestinal tract defects, absorption defects or digestive disorders, stress, alcoholism, smoking habits, etc. can also suffer from malnutrition. It is worth noting that sufficient food intake does not automatically mean sufficient intake of necessary nutritional factors, see [11] and [27].

According to Kleinwächterová and Brázdová [11], some causes of bad nutrition include: genetic and metabolic issues, exogenous factors linked to socio-economical status, nutrition (structure of nutrition, frequency of food intake, knowledge about nutrition, childhood food habits) and sports activities. According to recent research, biological leanness is related to hereditary factors, but fatness is not.

Kohout [12] and Urbánek [27] state that malnutrition can inhibit blood transportation, deteriorate muscles (the heart muscle reacts to malnutrition by weakening the active muscle matter of the myocardium), and causes shortness of breath, worsens gastrointestinal tract motility, decreases immunity, inhibits recovery, increases vulnerability to infectious complications, decreases the effectiveness of drugs, etc.

According to Rážová [20], the nourishment of the population in the Czech Republic has the following particular characteristics: unsuitable choice of food items (frequency, amounts, variety), high energetic intake, high intake of animal products (fats and proteins), bad ratio of nutrients in favour of saccharides (fibre), high intake of salt (smoked meat products), low intake of vegetable and fruit, bad water intake, etc.

This kind of bad nutrition causes various diseases, such as heart and vascular diseases, diabetes, intestinal cancers, etc., which often concern people living in economically developed countries and are characterized by overeating, sedentary lifestyle and stress. This is the reason why these diseases are called Civilizational. These diseases were uncommon in the past, see [11].

A regime of balanced nutrition combined with balanced energetic intake and necessary amounts of vitamins and minerals is generally accepted as having a protective effect. It constitutes the base for good health, quality of life as well as aiding in the prevention and treatment of many illnesses, see [12], [18], [20].

Nutrition from the perspective of linear programming is always about fulfilling all nutritional requirements of a larger group of people or the population from developing or industrial countries. The objective functions of the linear programming models are as follows: minimize climate impact through greenhouse gas emissions [4], [15]; minimize the difference between the optimal and current diet [4], [6], [16], [17]; minimize the cooking and preparation time of food [14]; and also minimize the cost [1], [3], [4], [7], [23]. The outcomes of the papers are recommendations or certain types of scenarios for people or government. To the author's best knowledge, there are no papers concerning the needs of an individual; not every person can follow the recommendations for the general population. Only specialists certified in healthcare nutrition can treat individuals, but these specialists lack efficient tools to prepare individually-orientated diets.

The model in this article reflects the methods adopted by a nutrition adviser and improves their processes as well as optimizes their effectiveness. It paves the way for modern nutritional consultancy. The model prevents some mistakes that the advisor could make when creating a feeding plan. The model takes the national recommendations for people in the Czech Republic into account. The model uses complete recipes including techniques of their preparation (the advisor prefers boiling and stewing). In our previous article [22], we worked with food items without technological processing only. The model prevents food wastage, takes into account the system of measurement of the client (pinch, teaspoon, spoon, cup, etc.), and it creates a more balanced eating plan.

The nutritional adviser always uses software, but the feeding plan is composed manually. The adviser has to choose the food items, follow the client's preferences (which food items the client does not like or cannot eat), follow the recommendations, etc. The new feeding plan must also be reasonable and has to be acceptable for the client. Licenced programs usually work with about 14 nutrients. It takes more time if the adviser works with more nutrients. That is why the adviser does not work with the majority of nutrients and the creating of the feeding plan is based on the adviser's experience and practice. Further details can be found in [22].

The below constructed model will greatly help the adviser. The adviser will be sure that the client obtains the best feeding plan, all the needs of the client are satisfied, the plan does not harm the client and the recipes are meaningful.

2 The Problem

The nutrition adviser offers individual consultations to two types of the clients: clients whose physician recommended them to visit the adviser (have some malnutrition, high blood pressure or have some diseases that can be affected by proper nutrition), and clients who are simply interested in a healthy style (want to fix some nutritional details, lose weight, need support in doing sport). In both cases, it is important to work with the client's physician.

The task of the adviser is to analyse the client's consumed food items and beverages, measure the client's body (weight, fat, etc.), to determine the individual nutritional values, and to compute the feeding plan. Then the adviser presents the plan to the client and compares it with the client's current eating habits, see [19] and [20].

The adviser's methodology of examination consists of two parts.

1) Diagnostic part – anamnesis

The client informs the adviser about the client's personal data, job (sedentary job, working environment, possibilities and style of feeding), intolerance, allergy, eating habits, smoking, alcohol, past and present illnesses (hypertension, diabetes mellitus, liver disease, etc.). Some physical and biochemical examination (cachexy, swelling, power of muscles, total cholesterol, LDL and HDL cholesterol, glycemia, etc.) are available from the physician's data. The adviser is also interested in the client's family anamnesis – if there are any genetical risks, for example high blood pressure, familial hypercholesterolemia, diabetes mellitus, heart attack before the age of 60, tumours and everything that should be taken into account when creating the client's diet. See [12] and [29] for further details.

2) Analytical part

This part includes the measurement of height, weight, BMI, circumference of limbs, hipline, waistline, measurement of subcutaneous fat, visceral fat, the amount of the muscle mass, the amount of minerals in the client's bones, blood pressure and the resting heart rate. Then the adviser evaluates the client's body composition and takes into account the measurements and other factors, such as psychological or social, when calculating the nutrient requirements.

Next, the client has to prepare a list of all food items consumed during at least one week before the meeting, including the amounts of the items, technology of preparation, time of eating and physical activities. For further details, see [11] and [12].

Then the adviser determines the *ideal body weight* as follows [12]

$$\omega_m = 0.655\, h_m - 44.1\,, \qquad (1)$$
$$\omega_f = 0.593\, h_f - 38.8\,,$$

where ω_m or ω_f is the ideal weight of a man or a woman, respectively, in kilograms and h_m or h_f is the height of the man or the woman, respectively, in centimetres.

The adviser recognizes the *basal energy expenditure* and the *total daily energy*.

The basal energy expenditure is important to support all functions of the body. We can determine the energy by using the *indirect calorimetry*. This technique uses the measurement of oxygen consumption and carbon dioxide expenditure when the client is breathing over a period of time. The equipment to perform the measurements is uncommon, see [27].

That is why the basal energy expenditure is determined by using the *Harris-Benedikt equation*. The equation was established experimentally by indirect calorimetry measurement of many people. The corresponding equations are as follows

$$\beta_m = 4.184(66.473 + 13.751\omega_m + 5.003h_m - 6.755\alpha_m)\,, \tag{2}$$
$$\beta_f = 4.184\big(655.095 + 9.563\omega_f + 1.849h_f - 4.675\alpha_f\big)\,,$$

where β_m or β_f is the basal energy of a man or a woman, respectively, in kilojoules (kJ) per day and α_m or α_f is the age of the man or the woman, respectively, in years. See [13] for different experimental calculations of basal energy.

When the adviser treats an obese client, the adviser has to use the *adjusted body weight* [27] instead of the ideal body weight in the Harris-Benedikt equation. This is due to the big difference between the current body weight and the ideal body weight, therefore the following is used

$$\omega' = 0.25\psi + \omega\,,$$

where ω' is the adjusted body weight, ψ is the real body weight and ω is the ideal body weight.

Apart from the basal energy expenditure, the *additional energy* corresponds to the demands made on the functioning of body activities including physical and psychological activity. According to [26], we can add it as follows.

We need to calculate the *factor of physical activity*. The calculation is generated from the list of the client's physical activities. It is calculated as the weighted average of relative times of activities performed by the client during a day; each activity has a specific weight (sleeping 0.95, resting 1.0, very easy work 1.5, hard work 7.0, see [12]). The relative time is the time (in hours) spent by the client to perform an activity divided by 24 hours. The weighted average is calculated for each day of the week and finally the average for the whole week is calculated. This one-week average is the factor of activity, denoted as ρ.

Then the total daily energy requirement can be calculated by using a device for monitoring the heart rate, or using the equation

$$\tau = \beta\rho + \delta, \tag{3}$$

where τ is the total daily energy requirement in kJ, β is the basal energy expenditure, ρ is the factor of activity and δ is the postprandial thermogenesis. (The postprandial thermogenesis of a healthy client is $\delta = 919$ kJ, see [2]).

According to [11], energy is taken from macronutrients, such as proteins, fats and saccharides. Micronutrients include vitamins and minerals. There are two classes of vitamins: fat-soluble (A, D, E, K) and water-soluble (the others).

Provazník [19] states that each nutrient is of a particular importance. For example, sodium is responsible for osmotic pressure balance; cholesterol is a building nutrient of bile acids and steroid hormones; magnesium is important to construct the bones and to decrease the nervous muscle tension. Fat-soluble vitamins are not excreted by urine, so the client can be overdosed. Every nutrient is needed in a certain amount.

We adjust the total amount of energy according to the higher heating value. The physical higher heating value is the amount of energy which is lost by completely burning one gram of a nutrient in a calorimetric bomb. One gram of saccharides yields 17 kJ of energy, one gram of proteins yields 23 kJ and lipids around 38 kJ. The values are distinct from the physiological higher heating values, which are the amounts of energy the body can utilize. In the case of saccharides and lipids, the values are almost the same, but in the case of proteins the physiological value is 16.7 kJ. The nutrients should be composed so that the 15%, 30%, and 55% of the total daily energy intake comes from proteins, fats, and saccharides, respectively, see [26].

3 Mathematical Model

The aim is to design the diet plan for some period of time, so let us consider $D = 7$ days (Monday, Tuesday, Wednesday, etc.) denoted by $d = 1, \dots, D$. There will be some meals during each of the days. We will work with $K = 5$ meals (breakfast, first snack, lunch, second snack, dinner) per day denoted by $k = 1, \dots, K$. So we will have 35 meals during the week in total. Every meal will be cooked according to some recipes, so let us consider recipes $r = 1, \dots, R$. A recipe is a set of instructions and food items that describes how to prepare a meal. So let us consider food items $j = 1, \dots, n$ (such as chicken, potatoes, cheese, milk, etc.), including drink items (such as tea, mineral water, juice, etc.).

The recipe r uses food items $S_r \subset \{1, \dots, n\}$, where $S_r \neq \emptyset$ and $\bigcup_{r=1}^{R} S_r = \{1, \dots, n\}$. Some of the sets S_r can be singletons. The recipes can be composed individually. That depends on the client's habits and the client's or the adviser's preferences. For example, if the client is a vegan, we can use recipes just for vegans from a recipe book.

Each food item consists of some nutrients, so let us consider nutrients $i = 1, \dots, m$ (such as fats, saccharides, proteins, ect.). Consider a real non-negative matrix $\boldsymbol{A} = (a_{ij})$ where a_{ij} means the quantity of nutrient i in one unit of the food item j for all $i = 1, \dots, m$ and for all $j = 1, \dots, n$. The aim is to satisfy the recommended daily intakes of nutrients, which should be between some upper and lower bound. Denote the minimal and maximal recommended daily intakes of all nutrients by a non-negative vector $\boldsymbol{b}^{\min} = (b_i^{\min})$ and a non-negative vector $\boldsymbol{b}^{\max} = (b_i^{\max})$, respectively, with $i = 1, \dots, m$.

If the client suffers from some nutrition malfunction or is in danger of certain illnesses, the vectors $\boldsymbol{b}^{\min}$ and $\boldsymbol{b}^{\max}$ have to be modified, i.e. the values of the recommended daily intakes of certain nutrients have to be increased, decreased or have to be equal to zero.

Let us have a binary matrix $\boldsymbol{C}^{RR} = (c_{r_1 r_2}^{RR})$ for all $r_1, r_2 = 1, \dots, R$ which mean compatibility between recipes ($c_{r_1 r_2}^{RR} = 1$ if recipes r_1 and r_2 are compatible, i.e. can be used in the same meal, and $c_{r_1 r_2}^{RR} = 0$ otherwise) and binary matrix $\boldsymbol{C}^{KR} = (c_{kr}^{KR})$ for all $r = 1, \dots, R$ and for all $k = 1, \dots, K$, which means compatibility between meal k and recipe r ($c_{kr}^{KR} = 1$ if meal k and recipe r are compatible, i.e., meal prepared according to the recipe r can be served in the meal k, and $c_{kr}^{KR} = 0$ otherwise). Clearly, the matrix $\boldsymbol{C}^{RR}$ will be symmetric and with ones on its diagonal.

Let us have real non-negative matrices $\boldsymbol{M}^{\min} = (m_{rj}^{\min})$ and $\boldsymbol{M}^{\max} = (m_{rj}^{\max})$ for all $r = 1, \dots, R$ and for all $j = 1, \dots, n$, which means the minimal and maximal quantity of the food item j in the recipe r. The elements will be positive, $0 < m_{rj}^{\min} \leq m_{jr}^{\max}$, if $j \in S_r$, and zero, $m_{rj}^{\min} = m_{rj}^{\max} = 0$, if $j \notin S_r$.

Now we can proceed with the formulation of the mathematical model. Let z_{dkr} be a binary variable which means if the recipe r is used in the meal k of the day d ($z_{dkr} = 1$) or not ($z_{dkr} = 0$). Two incompatible recipes cannot be used in the same meal. We can express that by the following inequalities

$$z_{dkr_1} + z_{dkr_2} \leq 1\,, \tag{4}$$

for all $d = 1, \dots, D$, for all $k = 1, \dots, K$ and for all $r_1, r_2 = 1, \dots, R$ such that $c_{r_1 r_2}^{RR} = 0$.

We also do not want to use the recipe r if it is not compatible with the meal k, so we use the condition

$$z_{dkr} = 0\,, \tag{5}$$

for all $d = 1, \dots, D$, for all $k = 1, \dots, K$ and for all $r = 1, \dots, R$ such that $c_{kr}^{KR} = 0$.

We introduce the real non-negative variabes x_{dkrj}. The vaule x_{dkrj} means the amount of the food item j used in the meal k and the recipe r in day d. We need to satisfy the client's minimal and maximal daily recommended intake as follows

$$\sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} a_{ij} x_{dkrj} \geq b_i^{\min}, \tag{6}$$

$$\sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} a_{ij} x_{dkrj} \leq b_i^{\max}, \tag{7}$$

for all $i = 1, \dots, m$ and for all $d = 1, \dots, D$. Inequalities (6) and (7) are typical constraints of the classical Diet problem.

We want to use reasonable amounts of food items in the recipes so we add inequalities

$$x_{dkrj} \geq m_{rj}^{\min} z_{dkr} , \tag{8}$$

$$x_{dkrj} \leq m_{rj}^{\max} z_{dkr} , \tag{9}$$

for all $d = 1, \ldots, D$, for all $k = 1, \ldots, K$, for all $r = 1, \ldots, R$ and for all $j \in S_r$.

There can be some nutrients which should be balanced in certain proportions. For example, according to [28], the ratio of the essential amino acids n-6:n-3 should be in the ratio 5:1. The proportion of the plant and animal proteins should be in the ratio 1:1, see [19]. Denote the set $I_1 = \{i_{11}, i_{12}, \ldots, i_{\iota\mu_1}\}$ of nutrients which should be in the ratio $\zeta_{11}:\zeta_{12}: \ldots :\zeta_{1\mu_1}$, set $I_2 = \{i_{21}, i_{22}, \ldots, i_{2\mu_2}\}$ of nutrients which should be in the ratio $\zeta_{21}:\zeta_{22}: \ldots :\zeta_{2\mu_2}$, etc., and set $I_\nu = \{i_{\nu 1}, i_{\nu 2}, \ldots, i_{\nu\mu_\nu}\}$ of nutrients which should be in the ratio $\zeta_{\nu 1}:\zeta_{\nu 2}:\ldots:\zeta_{\nu\mu_\nu}$. The nutrients can be in the ratios with some tolerances. Let $\varepsilon_{\iota\kappa}$ be the tolerance of the coefficient $\zeta_{\iota\kappa}$ for $\iota = 1, \ldots, \nu$ and $\kappa = 1, \ldots, \mu_\iota$. We assume that $0 < \varepsilon_{\iota\kappa} < \zeta_{\iota\kappa}$.

We can express that as follows

$$\begin{aligned} (\zeta_{\iota\kappa} - \varepsilon_{\iota\kappa}) \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} a_{i_{\iota\lambda} j} x_{dkrj} &\leq (\zeta_{\iota\lambda} + \varepsilon_{\iota\lambda}) \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} \\ (\zeta_{\iota\kappa} + \varepsilon_{\iota\kappa}) \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} a_{i_{\iota\lambda} j} x_{dkrj} &\geq (\zeta_{\iota\lambda} - \varepsilon_{\iota\lambda}) \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in S_r} \end{aligned} \tag{10}$$

(11)

for all $\iota = 1, \ldots, \nu$, for all $\kappa = 1, \ldots, \mu_\iota - 1$, for all $\lambda = \kappa + 1, \ldots, \mu_\iota$ and for all $d = 1, \ldots, D$.

In some situations, the variable x_{dkrj} should attain discrete values. For example, if $j = j_0$ is an egg of medium size, then the variables x_{dkrj_0} should be integer (the number of eggs). Or the client may use a system of measurement involving pinch (0.5 grams), cups (Figure 1 and Figure 2) with discrete cup system or spoons (see Figure 3) with discrete system of measurement. Then the variable x_{dkrj} should also be discrete. So denote $J \subseteq \{1, \ldots, n\}$ the set of food items such that the corresponding variables x_{dkrj} should be integer. Then $x_{dkrj} \in \mathbb{Z}$ for all $j \in J$, for all $d = 1, \ldots, D$, for all $k = 1, \ldots, K$ and for all $r = 1, \ldots, R$.

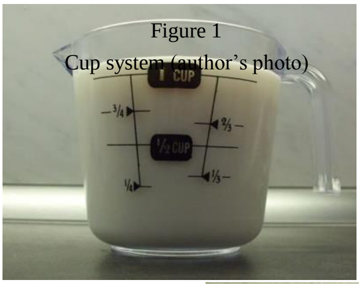

Figure 1

Cup system (author's photo)

Figure 3

Spoon system (author's photo)

We would like to avoid the situation of eating too much or too little in some meals. Denote the minimal and maximal energy intake as $b_1^{\min}$ and $b_1^{\max}$. Inspired by [21], we can naturally distribute the energy intake during the whole day among the meals, for example 20% of the total daily energy for breakfast, 12.5% for the first snack, 30% for lunch, 12.5% for the second snack and 25% for dinner. This depends on the feeding plan which the adviser is preparing. The desired energy intake distribution during the day is given by the non-negative vector $\boldsymbol{v}$=(v_k) with $\sum_{k=1}^{K} v_k = 1$. Then we want to satisfy the inequalities

$$\sum_{r=1}^{R} \sum_{j \in S_r} a_{1j} x_{dkrj} \geq v_k \, b_1^{\min}, \tag{12}$$

$$\sum_{r=1}^{R} \sum_{j \in S_r} a_{1j} x_{dkrj} \leq v_k \, b_1^{\max}, \tag{13}$$

for all $d = 1, \dots, D$ and for all $k = 1, \dots, K$, where the nutrient no. 1 is energy.

The advisers do not usually care about wasting the food. We know that some food items are bought in packages of specific sizes. For example, we can buy yoghurt in packages of 100, 150, 200 or 500 grams, or eggs in packages of 6, 10, 15, 20 or 30 pieces.

Denote the set $\Xi = \{j_1, \dots, j_\vartheta\}$ of food items which are bought in packages of specific sizes. Let food item j_1 be bought in packages of sizes $\mathrm{P}_{j_1}^1, \mathrm{P}_{j_1}^2, \dots, \mathrm{P}_{j_1}^{\theta_{j_1}}$, where $0 < \mathrm{P}_{j_1}^1 < \mathrm{P}_{j_1}^2 < \cdots < \mathrm{P}_{j_1}^{\theta_{j_1}}$, let food item j_2 be bought in packages of sizes $\mathrm{P}_{j_2}^1, \mathrm{P}_{j_2}^2, \dots, \mathrm{P}_{j_2}^{\theta_{j_2}}$, where $0 < \mathrm{P}_{j_2}^1 < \mathrm{P}_{j_2}^2 < \cdots < \mathrm{P}_{j_2}^{\theta_{j_2}}$, etc., and let food item j_ϑ be bought in packages of sizes $\mathrm{P}_{j_\vartheta}^1, \mathrm{P}_{j_\vartheta}^2, \dots, \mathrm{P}_{j_\vartheta}^{\theta_{j_\vartheta}}$, where $0 < \mathrm{P}_{j_\vartheta}^1 < \mathrm{P}_{j_\vartheta}^2 < \cdots < \mathrm{P}_{j_\vartheta}^{\theta_{j_\vartheta}}$. For $j \in \Xi$, we can use equations like

$$\sum_{d=1}^{D} \sum_{k=1}^{K} \sum_{\substack{r=1 \\ S_r \ni j}}^{R} x_{dkrj} = \sum_{\pi=1}^{\theta_j} (\mathrm{P}_j^{\pi} - \mathrm{P}_j^{\pi-1})\, \xi_j^{\pi}, \tag{14}$$

for all $j \in \Xi$, where ξ_j^π are new integer variables such that $0 \le \xi_j^1 \le \xi_j^2 \le \cdots \le \xi_j^{\theta_j}$. For all $j \in \Xi$, we put $\mathrm{P}_{j0} = 0$.

Example: Food item j can be bought in packages of 100 grams, 150 grams and 180 grams, so let $\mathrm{P}_{j0} = 0$, $\mathrm{P}_{j1} = 100$, $\mathrm{P}_{j2} = 150$ and $\mathrm{P}_{j3} = 180$. Then we can use equations like

$$\sum_{d=1}^{D} \sum_{k=1}^{K} \sum_{\substack{r=1 \\ S_r \ni j}}^{R} x_{djkr} = 100\, \xi_j^{100} + 50\, \xi_j^{150} + 30\, \xi_j^{180},$$

where ξ_j^{100}, ξ_j^{150}, ξ_j^{180} are new integer variables such that $0 \le \xi_j^{100} \le \xi_j^{150} \le \xi_j^{180}$. The coefficient 30 by ξ_j^{180} means the difference between the size of the packages of 150 and 180 grams and 50 by ξ_j^{150} the difference between the size of the packages of 100 and 150 grams. We formulate analogous equations for each food item $j = 1, \dots, n$ that is supplied in packages.

To exclude the situation when some recipes are repeating during the week, we add inequalities

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} z_{dkr} \le 1\,, \tag{15}$$

for all $d_0 = 1, \dots, D-6$ and for all $r = 1, \dots, R$.

The adviser should follow the national nutrition recommendations for the population. According to Dostálová [5] and Hrnčířová [9], there are specific recommendations for people from the Czech Republic; what and how much they should eat or drink, including the recommendations about the intake of nutrients:

- some fermented food items every day,
- legumes at least two times a week,

- lean meat (300–400 grams) every week and a combination of poultry and veal,
- fish (400 grams) at least twice a week,
- animal viscera (liver, lungs, stomach, etc.) once every two weeks,
- handful of nuts a day (10 grams)
- 3 or 4 eggs a week
- vegetables at least 400 grams a day
- fruit 150–200 grams a day
- food of plant origin at least once a week [19]
- water intake at least 22 mililitres per 1 kilogram of personal weight where all minerals from mineral water should be between 150–500 miligrams per litre,
- alcohol: men wine/beer/spirits at most 250/500/60 mililitres, respectively, women at most 125/300/40 mililitres, respectively,
- sweets, smoked meat and other salted products eaten rarely,
- etc.

If the recipe r is used, then the corresponding food items $j \in S_r$ which the recipe consists of must be used. For that reason we introduce new binary variables y_{dkj} which mean whether the food item j is used in the meal k of the day d ($y_{dkj} = 1$) or not ($y_{dkj} = 0$). So we require that

$$y_{dkj} \geq z_{dkr} , \tag{16}$$

for all $d = 1, \dots, D$, for all $k = 1, \dots, K$, for all $r = 1, \dots, R$ and for all $j \in S_r$.

Conversely, if we use the food item j, then at least one recipe r including this food item must be used. Inequalities to express this condition are as follows

$$y_{dkj} \leq \sum_{\substack{r=1 \\ S_r \ni j}}^{R} z_{dkr} , \tag{17}$$

for all $d = 1, \dots, D$, for all $k = 1, \dots, K$ and for all $j = 1, \dots, n$.

We introduce sets $E_1, E_2, \dots, E_T$ of food items that are related as above, for example set E_1 of fermented food items, set E_2 of legumes, etc.

So let us consider the set of all fermented food items E_1. We know that the client should eat or drink some fermented food items every day. We express this condition by inequalities

$$\sum_{k=1}^{K} \sum_{j \in E_1} y_{dkj} \geq 1 , \tag{18}$$

for all $d = 1, \dots, D$. We should eat legumes from the set E_2 at least two times a week so we introduce the inequalities

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} \sum_{j \in E_2} y_{dkj} \geq 2\,, \tag{19}$$

for all $d_0 = 1, \dots, D-6$.

We should eat 300–400 grams of lean meat from the set E_3 every week and combine poultry and veal from E_{31} and E_{32} where the sets E_{31}, E_{32} are disjoint and $E_{31} \cup E_{32} \subseteq E_3$. This condition is expressed by inequalities

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in E_3} x_{dkrj} \geq 300\,, \tag{20}$$

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} \sum_{r=1}^{R} \sum_{j \in E_3} x_{dkrj} \leq 400\,, \tag{21}$$

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} \sum_{j_1 \in E_{31}} y_{dkj_1} \geq 1\,, \tag{22}$$

$$\sum_{d=d_0}^{d_0+6} \sum_{k=1}^{K} \sum_{j_2 \in E_{32}} y_{dkj_2} \geq 1\,, \tag{23}$$

for all $d_0 = 1, \dots, D-6$.

Furthermore, the client should eat food of plant origin, i.e. exclude meat during the whole day, at least once a week. So let the set E_4 include all meat items and let us add inequalities

$$\sum_{k=1}^{K} \sum_{j \in E_4} y_{dkj} \leq K\,|E_4|\,\eta_d\,, \tag{24}$$

$$\sum_{d=d_0}^{d_0+6} \eta_d \leq 6\,, \tag{25}$$

for all $d = 1, \dots, D$ and for all $d_0 = 1, \dots, D-6$, where $|E_4|$ is the cardinality of the set E_4 and η_d are new binary variables.

Inequalities for the rest of the recommendations are analogous to (18)–(25).

Finally, we consider fluid intake. According to Zavadilová [28], adults should drink 22–38 mililitres of water per one kilogram of body weight in the weather with temperature between 22–37°C every day. The recommended fluid intake also depends on physical activity. Let E_T be the set of sparkling water, tea, juice, mineral water and other liquids. Then we add inequalities

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{j\in E_T} x_{dkrj} \geq b_m^{\min}, \tag{26}$$

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\sum_{j\in E_T} x_{dkrj} \leq b_m^{\max}, \tag{27}$$

for every $d = 1, \dots, D$, where $b_m^{\min}$ or $b_m^{\max}$ is minimal or maximal amount of fluids per day, respectively.

The above conditions correspond to the recommendations of diet for the Czech Republic and the adviser can apply them to the clients who prefer a balanced diet plan. Of course, if the client cannot eat some food item, we do not include the food item into the sets $E_1, E_2, \dots, E_T$.

The entire model is a mixed-integer linear programming model and consists of contraints (4)–(27). The model should use a large database of recipes. We can add an objective function (minimize the price of eaten food items or minimize the difference between the current and new bought food items) to the model, but this is not necessary for us now. We need to only find a feasible solution, so we minimize the zero objective function.

4 Results

Let us describe a particular client. We will not show the whole anamnestic and analytical part described in Section 2, but only the necessary fundamental data that we need to show our results of the model in Section 3.

The client is a woman, 26 years old and 173 cm tall. Using the equation (1) from Section 2 we have the ideal body weight $\omega_f = 64$ kilograms. Then we can calculate from equation (2) the basal energy expenditure $\beta_f = 6166$ kJ and from (3) the total energy $\tau = 10109$ kJ. Using the anamnestic and analytical part we can determine the amounts of the macro- and micronutrients. The macronutrients are as follows: 89 or 82 or 327 grams of proteins or fats or saccharides, respectively. The exact values can be in the tolerance of 5% so we can work with intervals. The amounts of microelements are simply inspired by [8], [24] and [25].

Table 1 presents the amounts of nutrients contained in food items we work with, the minimal and maximal recommended amounts of nutrients and a solution for one day in the Solution column. In the calculations we used 70 food items in 40 recipes and 31 nutrients.

In total, there are 43564 variables, out of which 22340 are integer, and 397642 constraints in the model.

Table 1
Contains the input data, calculated amounts of nutrients and final result per day

Nutrient	**Food items [100 g]**			**Recommended amounts**		
	Chicken	**Potato**	**…**	**Minimum**	**Solution**	**Maximum**
Energy [kJ]	694	322	…	9098	11006	11122
Proteins [g]	20	2	…	80	97	98
Fats [g]	10	0	…	73	80	90
Saccharides [g]	0	16	…	294	330	360
Fibre [g]	0	2	…	25	25	35
⋮	⋮	⋮	…	⋮	⋮	⋮
Vitamin [C]	2	15	…	75	220	230
⋮	⋮	⋮	⋮	⋮	⋮	⋮

The food plan found by our model for one day is presented in detail in Table 2.

Table 2
Optimal diet plan for one day

Meal	**Food items**
Breakfast	250 ml milk, 45 g oat flakes, 10 g almonds, 30 g orange, 7 g honey
Snack	100 g curd, 60 g apple, 30 g orange, 5 g linseed oil
Lunch	Soup: 75 g whole-wheat pasta, 30 g sweet corn, 25 g peas, 200 ml broth The main course: 130 g lentils, 150 g chicken, 5 g sunflower oil Salad: 25 g cucumber, 25 g potato, 35 g iceberg lettuce, 3 g olive oil, 1 pinch sesame seeds
Snack	60 g kaiser rolls (1 piece), 8 g margarine, 60 g cheese
Dinner	150 g slice a bread, egg spread (1 egg, 10 g margarine, 35 g curd, 3 g chives) Salad: 25 g tomato, 100 g bell pepper, 50 g cucumber
Drinking	250 ml fresh orange juice and 250 ml water, 2000 ml unsweetened tea

This model was solved by the optimization software FICO® Xpress Optimization Suite on a Windows XP SP3 computer with 1 GB RAM and Intel Atom 1.60 GHz CPU. The computation took about 45 seconds.

5 Discussion

The nutritional adviser is unable to create a diet for the whole week that respects the optimal amounts of all 31 nutrients every day.

If the adviser decides to use the presented mathematical model, then the adviser's work is reduced to assigning of food items to the recipes, assigning the recipes to the meals and setting minimal and maximal amounts of nutrients.

We would like to extend the model so that it includes not only the client but also the family members of the client. The reason is that, in practice, it is not easy to prepare different meals daily for everybody. And so in many cases it can happen that the client does not manage the diet properly or stops following the nutrition recommendations all together.

The model can also be extended to nutrition healthcare in hospitals and be useful for nutritional assistants, nutritional therapists and nutritionists (physicians specialized in artificial nutrition).

Conclusions

This article provides a new approach that will help improve the effectiveness for the nutritional adviser. We introduced a tool, which efficiently optimizes the adviser’s work. It saves the adviser’s time and effort, not only by supporting one client, but by supporting all the adviser’s clients.

Acknowledgement

The author would like to thank Dr. David Bartl for useful discussions and comments which helped to improve the paper. This research was supported by the internal grant no. SGS12/PřF/2016–2017, Geometric mechanics, optimization and number theory. The use of the FICO® Xpress Optimization Suite under the FICO Academic Partner Program is gratefully acknowledged.

References

[1] Anderson, M., A., Earle, D., M.: Diet Planning in the Third World by Linear and Goal Programming. *The Journal of the Operational Research Society* **34.1** (1983) 9-16

[2] Bender, D., A.: *Introduction to Nutrition and Metabolism.* CRC Press, London, 2002

[3] Conforti, P., D’Amicis, A.: What is the cost of a healthy diet in terms of achieving RDAs? *Public Health Nutrition* **3.3** (2000) 367-373

[4] van Dooren, C., et al.: Combining Low Price, Low Climate Impact and High Nutritional Value in One Shopping Basket through Diet Optimization by Linear Programming. *Sustainability* **7.9** (2015) 12837-12855

[5] Dostálová, J., Dlouhý, P., Tláskal, P.: *Výživová doporučení pro obyvatelstvo České republiky* [on-line]. Společnost pro výživu, 2012, Available from: http://www.vyzivaspol.cz/vyzivova-doporuceni-pro-obyvatelstvo-ceske-republiky/ (cited 6 July 2016)

[6] Ferguson, E., L., et al.: Food-based dietary guidelines can be developed and tested using linear programming analysis. *The Journal of Nutrition* **134.4** (2004) 951-957

[7] Foytik, J.: Very low-cost nutritious diet plans designed by linear programming. *Journal of Nutrition Education* **13.2 (**1981) 63-66

[8] Heseker, H., Heseker, B.: *Die Nährwerttabelle: [über 40.000 Nährstoffangaben; einfache Handhabung; Tabellen zu Laktose, Fruktose, Gluten, Purin, Jod und trans-Fettsäuren]*. Umschau, Neustadt and der Weinstraße, 2016

[9] Hrnčířová, D., Rambousková, J., et al.: *Výživa a zdraví* [on-line]. Ministerstvo zemědělství, Odbor bezpečnosti potravin, Praha, 2012, Available from: http://www.bezpecnostpotravin.cz/UserFiles/publikace/ Vyziva_a_zdravi.pdf (cited 16 August 2016)

[10] Kastnerová, M.: *Poradce pro výživu.* [*Nutrition Adviser*. In Czech.] Nová Forma, České Budějovice, 2011

[11] Kleinwächterová, H., Brázdová, Z.: *Výživový stav člověka a způsoby jeho zjišťování.* [*Nutritional Status of People and Methods of its Detection.* In Czech.] Institut pro další vzdělávání pracovníků ve zdravotnictví v Brně, Brno, 2001

[12] Kohout, P.: *Dokumentace a hodnocení nutričního stavu pacientů.* [*Documentation and Evaluation of Nutritional Status of Patients.* In Czech] Forsapi, Praha, 2011

[13] Legge, A.: How to Estimate Your Maintenace Calories. *Complete Human Performance* [on-line] Available from: http://www.completehumanperformance.com/calorie-needs/ (cited 28 August 2016)

[14] Leung, P., Wanitprapha, K., Quinn, L., A.: A recipe-based, diet-planning modelling system. *British Journal of Nutrition* **74.2** (1995) 151-162

[15] Macdiarmid, J., et al.: Sustainable diets for the future: can we contribute to reducing greenhouse gas emissions by eating a healthy diet? *The American Journal of Clinical Nutrition* **96.3** (2012) 632-639

[16] Maillot, M., Drewnowski, A.: Energy allowances for solid fats and added sugars in nutritionally adequate US diets estimated at 17-33% by a linear programming model. *The Journal of Nutrition* **141.2** (2011) 333-340

[17] Okubo, H., et al.: Designing optimal food intake patterns to achieve nutritional goals for Japanese adults through the use of linear programming optimization models. *Nutrition Journal* **14.57** (2015) 1-10

[18] Provazník, K., Komárek, L.: *Manuál prevence v lékařské praxi, VII. Doporučené preventivní postupy v primární péči.* [*Prevention Manual in Medicine Practice, VII. Recommended Preventive Procedures in Primal Healthcare*. In Czech.] Fortuna, Praha, 1999

[19] Provazník, K., Komárek, L., Janovská, J., Ošancová, K.: *Manuál prevence v lékařské praxi, II. Výživa.* [*Prevention Manual in Medicine Practice, II. Nutrition.* In Czech.] Fortuna, Praha, 1995

[20] Rážová, J.: *Metody a postupy v poradnách podpory zdraví.* [*Methods and Procedures in Health Care Advisory Work.* In Czech] Centrum zdraví Praha, Praha, 2001

[21] Reihserová, R.: *Jak poskládat stravu v průběhu dne?* [on-line]. Svět potravin. Available from: http://www.svet-potravin.cz/clanek.aspx?id=4177 (cited 7 July 2016)

[22] Schaynová, L.: A Client's Health from the Point of View of the Nutrition Adviser using Operational Research. In *Proceedings of the 34th International Conference on Mathematical Methods in Economics* 2016, Liberec: Technical University of Liberec (2016) 751-756

[23] Sklan, D., Dariel, I.: Diet planning for humans using mixed-integer linear programming. *British Journal of Nutrition* **70.1** (1993) 27-35

[24] Společnost pro výživu: *Referenční hodnoty pro příjem živin.* [*Dietary Reference Intakes for Nutrients.* In Czech] Výživa a servis s.r.o., Praha, 2011

[25] Svačina, Š., et al.: *Klinická dietologie.* [*Clinical Nutrition.* In Czech] Grada, Praha, 2008

[26] Trojan, S., Kittnar, O., Koudelová, J., et al.: *Lékařská fyziologie.* [*Medical Physiology.* In Czech] Avicenum, 1994

[27] Urbánek, L., Urbánková, P., et al.: *Klinická výživa v současné praxi.* [*Nutrition in Current Clinical Practice.* In Czech] Národní centrum ošetřovatelství a nelékařských zdravotnických oborů, Brno, 2008

[28] Zavadilová, V.: *Výživa a zdraví*. [*Nutrition and Health.* In Czech] Ostravská univerzita v Ostravě, Ostrava, 2014

[29] Zlatohlávek, L., et al.: *Klinická dietologie a výživa.* [*Clinical Nutrition and Dietetics.* In Czech] Current Media, Praha, 2016