Modeling of Stress-Strain State of Road Covering with Cracks

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Abstract: The stress-strain state of road covering in the course of operation is considered. It is assumed that the cross section of the covering has arbitrary number of rectilinear cracks. Force interaction of the wheel (roller) and road covering with rough upper surface is investigated. Using the perturbation method and the method of singular integral equations the contact problem of the pressing of the wheel (roller) in the road surface was solved. The stress intensity factors for the vicinity of the cracks vertices are found.

Keywords: road covering; elastic base; rectilinear cracks; stress intensity factors; rough surface

1 Introduction

Timely detection of various damages of road covering is of particular importance for providing reliable and safe functioning of road transport. In this connection the defects as cracks are of significant interest. Setting of the norms of admissible presence of defects, choice of the methods and periodicity of defectoscopic control of road is an important problem for increasing durability of road covering. While evaluating durability of road covering of motor roads it is necessary to proceed from possibility of presence of the most dangerous unrevealed defects in coverings. In this connection, the initial defects should be accepted to be equal to sensitivity limit of the used defectoscopic device.

Real surfaces of roads differ by the presence of roughnesses that are the unavoidable consequence of technological process. In spite of smablness of geometric distortions in the form of surface roughness, their role in friction, wear and fracture and etc. is very great [1-3]. Therefore, investigation of the roughness geometry itself for strength and the relation of roughness with the characteristics of physical-technical phenomena (friction, wear, fracture) generated by it are very significant. In this connection development of design models of investigation of parameters of road covering fracture is a very urgent problem [4-12].

2 Formulation of the Problem

Consider the stress-strain state of road covering during operation process. Let the cross section of the road covering have N rectilinear internal cracks of length $2l_k$ (k=1,2,...,N) (Fig. 1). It is assumed that the cracks are open and not filled.

For calculating the stress-strain state of the road covering near the rolling surface, in this case we arrive at the following contact problem of fracture mechanics.

Let us consider force interaction of the wheel and covering. Taking into account that the sizes of the contact area while contacting with covering are small compared with typical linear size of road covering in the plan, in the statement and solution of the contact problem the covering may be replaced by an elastic strip of thickness h situated on an elastic base in the form of elastic half-plane.

We model the material of the covering by an elastic medium with mechanical characteristics E_1 (elasticity modulus), μ_1 (Poisson ratio). Accordingly, we model the elastic base by elastic medium with mechanical characteristics E_2, μ_2. As a rule, the external surface of road covering has roughnesses of rolling surface.

Let us consider the following contact problem for an elastic strip with elastic base in the form of a half-plane. A wheel under the arbitrary system of forces is pressed into an elastic strip with internal cracks and rough upper surface. We can assume that normal force P_k (clip force) and moment M is applied to each unit of the length of the contact area. The base of the hard wheel is characterized by a rather smooth function $f_*(x)$.

It is required to determine the laws of contact stress and stress intensity factors distribution in the vicinity of the cracks tips.

Denote by $q(x)$ and $\tau(x)$ normal and tangential stresses, respectively, applied to the boundary of the half-plane (base of the covering). Denote the wheel's pressure on the covering by $p(x)$, the segment $[a_1,a_2]$ will be the contact area. In addition to normal forces (pressure) $p(x) = -\sigma_y(x,0)$, the tangential stresses $\tau_{xy}(x,0)$ connected with contact pressure by the Amonton-Coulomb law

$$\tau_{xy} = f\,p(x)$$

where f is the friction factor of the pair wheel-road covering, are also act in the contact area $a_1 \leq x \leq a_2$.

Consider some realization of the roughness of the external surface of the road rolling L_1'. Represent the boundary of the external contour L_1', in the form

$$y = \delta(x)$$

We will assume the contour L_1' close to the rectilinear form assuming only small deviations of the line L_1 from the straight line $y = 0$.

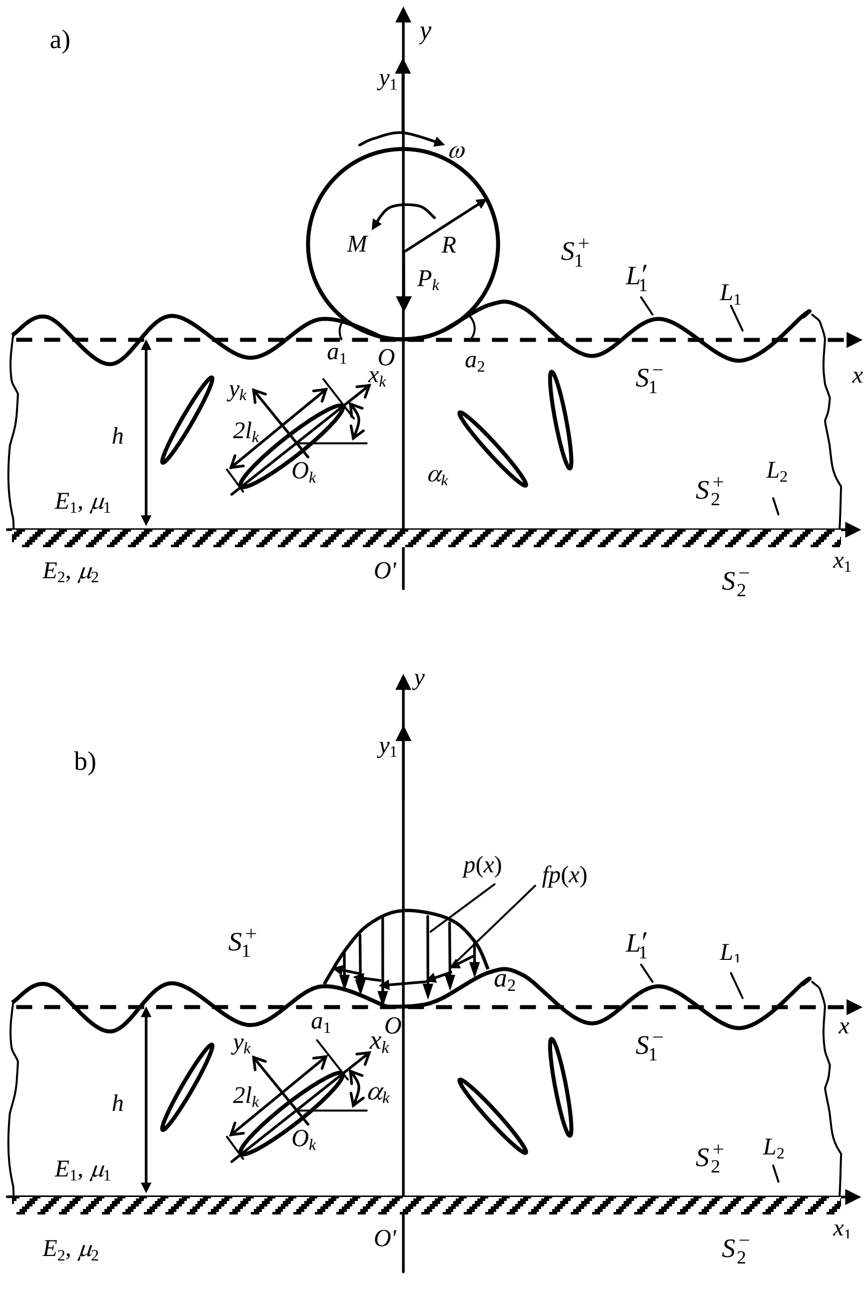

Figure 1

Calculation scheme of a contact fracture mechanics problem

On the base what has been said above, we write the boundary conditions of the considered contact problem of fracture mechanics in the form

for $y = \delta(x)$ $\sigma_n = 0$, $\tau_{nt} = 0$ exterior to the contact area (1)

for $y = \delta(x)$ $v_n = f(x) + \alpha x + C$, $\tau_{nt} = f\sigma_n$ on the contact area (2)

for $y = -h$ $(\sigma_y - i\tau_{xy})_{\mathrm{I}} = (\sigma_y - i\tau_{xy})_{\mathrm{II}}$, $(u+iv)_{\mathrm{I}} = (u+iv)_{\mathrm{II}}$ (3)

$\sigma_n = 0$; $\tau_{nt} = 0$ on the cracks faces

Here it is accepted that in the external surface area of the covering where the wheel is pressed, the dry friction forces occur; exterior to the contact area the surface of covering is free from external forces. The cracks faces are free from external loads. Stresses and displacements (perfect coupling conditions) are equal on the interphase of medium (covering and elastic base); $i = \sqrt{-1}$ is an imaginary unit; C is the translation of penetration (wheel), α is a turning angle of the penetrator; $f(x) = f_*(x) + \delta(x)$. Furthermore, the following additional conditions hold:

$$P_k = \int_{a_1}^{a_2} p(t)\,dt\,, \qquad M = \int_{a_1}^{a_2} t\,p(t)\,dt \tag{4}$$

3 The Case of One Crack

As it was accepted that the functions $\delta(x)$ and $\delta'(x)$ are small quantities, we can write the equation of the upper contour of the covering as follows:

$$y = \delta(x) = \varepsilon H(x) \tag{5}$$

where ε is a small parameter for which we can accept the greatest height of the roughness of the upper surface of the road covering related to the thickness of the covering.

Expand the stress tensor components σ_x, σ_y, τ_{xy} in series in small parameter of ε

$$\sigma_x = \sigma_x^{(0)} + \varepsilon\sigma_x^{(1)} + \ldots, \qquad \sigma_y = \sigma_y^{(0)} + \varepsilon\sigma_y^{(1)} + \ldots, \qquad \tau_{xy} = \tau_{xy}^{(0)} + \varepsilon\tau_{xy}^{(1)} + \ldots \tag{6}$$

Expanding in series the expressions for the stresses in the vicinity $y = 0$, we find the values of the stress tensor components for $y = \delta(x)$.

Using the perturbations method, allowing what has been said, we get the following conditions: for the covering in a zero approximation for $y = 0$ $\sigma_y^{(0)} = 0$, $\tau_{xy}^{(0)} = 0$ exterior to the contact area (7)

$\sigma_y^{(0)} = -p^{(0)}(x)$, $\tau_{xy}^{(0)} = -fp^{(0)}(x)$ on the contact area

$\sigma_n^{(0)} = 0$, $\tau_{nt}^{(0)} = 0$ on the cracks faces (8)

for $y = -h$ $\sigma_y^{(0)} = q^{(0)}(x)$, $\tau_{xy}^{(0)} = \tau^{(0)}(x)$ (9)

for the covering in a first approximation

for $y = 0$ $\sigma_y^{(1)} = N$, $\tau_{xy}^{(1)} = T$ exterior to the contact area (10)

$\sigma_y^{(1)} = N - p^{(1)}(x)$, $\tau_{xy}^{(1)} = T - fp^{(1)}(x)$ on the contact area

$\sigma_n^{(1)} = 0$, $\tau_{nt}^{(1)} = 0$ on the cracks faces (11)

for $y = -h$ $\sigma_y^{(1)} = q^{(1)}(x)$, $\tau_{xy}^{(1)} = \tau^{(1)}(x)$ (12)

for elastic base in a zero approximation

for $y = -h$ $\sigma_y^{(0)} = q^{(0)}(x)$, $\tau_{xy}^{(0)} = \tau^{(0)}(x)$ (13)

in a first approximation

for $y = -h$ $\sigma_y^{(1)} = q^{(1)}(x)$, $\tau_{xy}^{(1)} = \tau^{(1)}(x)$ (14)

Here $$N = 2\tau_{xy}^{(0)}\frac{d\Delta}{dx} - H\frac{\partial\sigma_y^{(0)}}{\partial y}, \quad T = \left(\sigma_x^{(0)} - \sigma_y^{(0)}\right)\frac{dH}{dx} - H\frac{\partial\tau_{xy}^{(0)}}{\partial y}, \tag{15}$$

the quantities N and T are known on the base of zero solution $\sigma_x^{(0)}$, $\sigma_y^{(0)}$, $\tau_{xy}^{(0)}$ and the function $H(x)$ describing the rough contour of the upper surface of road covering.

Because of smallness of the small parameter ε, in what follows we will be restricted in expansions (6) by the terms to the first order of smallness inclusively, with respect to ε.

Knowledge on the stress intensity factor allowing in the considered case to investigate the ultimate state of road covering and their durability on their base is of significant interest for predicting fracture.

According to perturbations method, the stress intensity factors for the vicinity of the cracks tip are found as follows

$$K_{\mathrm{I}} = K_{\mathrm{I}}^{(0)} + \varepsilon K_{\mathrm{I}}^{(1)} + \ldots, \qquad K_{\mathrm{II}} = K_{\mathrm{II}}^{(0)} + \varepsilon K_{\mathrm{II}}^{(1)} + \ldots$$

Here $K_{\mathrm{I}}^{(0)}$, $K_{\mathrm{II}}^{(0)}$ are the stress intensity factors for a zero approximation, $K_{\mathrm{I}}^{(1)}$, $K_{\mathrm{II}}^{(1)}$ for a first approximation, respectively.

In the center of the rectilinear crack locate the origin of the local system of coordinates $x_1O_1y_1$ whose axis x_1 coincides with the linear crack and forms the angle α_1 with the axis x (Fig. 1). The stress-strain state of road covering, at each approximation satisfies the system of differential equations of plane theory of elasticity.

Use the superposition principle. Then we can represent the stress and strain state of a two-layer body with a crack in the form of the sum of two states. The first state will be determined from the solution of contact problem (1)-(3) for a two-layer body in unavailability of a crack. The second state is determined from the solution of a boundary value problem for a cracked covering with forces on the faces determined by the first stress state. The first state for each approximation in unavailability of a crack is known [13].

The boundary conditions of the second problem are of the form:

in a zero approximation

$$\text{for} \quad y_1 = 0 \quad \sigma_{y_1}^{(0)} = -p_*^{(0)}(x_1), \quad \tau_{x_1y_1}^{(0)} = -p_1^{(0)}(x_1) \quad \left(|x_1| \le l_1\right) \tag{16}$$

$$\text{for} \quad y = 0 \quad \sigma_y^{(0)} = 0, \quad \tau_{xy}^{(0)} = 0$$

$$\text{for} \quad y = -h \quad \sigma_y^{(0)} = 0, \quad \tau_{xy}^{(0)} = 0 \quad \left(|x| < \infty\right) \tag{17}$$

in a first approximation

$$\text{for} \quad y_1 = 0 \quad \sigma_{y_1}^{(1)} = -p_*^{(1)}(x_1), \quad \tau_{x_1y_1}^{(1)} = -p_1^{(1)}(x_1) \quad \left(|x_1| \le l_1\right) \tag{18}$$

$$\text{for} \quad y = 0 \quad \sigma_y^{(1)} = 0, \quad \tau_{xy}^{(1)} = 0$$

for $y = -h$ $\sigma_y^{(1)} = 0$, $\tau_{xy}^{(1)} = 0$ $(|x| < \infty)$ (19)

Here $p_*^{(0)}(x_1)$, $p_1^{(0)}(x_1)$ and $p_*^{(1)}(x_1)$, $p_1^{(1)}(x_1)$ are normal and tangential stresses arising in continuous covering along the axis x_1 in zero and first approximations, respectively, from the application of the given loads relieving stress on the covering boundary. The quantities $p_*^{(0)}(x_1)$, $p_1^{(0)}(x_1)$ and $p_*^{(1)}(x_1)$, $p_1^{(1)}(x_1)$ are determined from the relations of [13]. The boundary conditions of problem (16)-(17) are written by means of Kolosov-Muskheleshvili formulas [14] in the form of a boundary value problem for finding two analytic functions $\Phi(z)$ and $\Psi(z)$

for $y = 0$ $\Phi_0(z) + \overline{\Phi_0(z)} + z\overline{\Phi_0'(z)} + \overline{\Psi_0(z)} = 0$ (20)

for $y = -h$ $\Phi_0(z) + \overline{\Phi_0(z)} + z\overline{\Phi_0'(z)} + \overline{\Psi_0(z)} = 0$

for $y_1 = 0$ $\Phi_0(x_1) + \overline{\Phi_0(x_1)} + x_1\overline{\Phi_0'(x_1)} + \overline{\Psi_0(x_1)} = f^{(0)}(x_1)$,

where $f^{(0)}(x_1) = -\left(p_*^{(0)}(x_1) - ip_1^{(0)}(x_1)\right)$.

We will seek the complex potentials $\Phi_0(z)$ and $\Psi_0(z)$ in the form [15]

$$\Phi_0(z) = \frac{1}{2\pi}\sum_{k=0}^{2}\int_{-l_k}^{l_k}\frac{g_k^0(t)dt}{t - z_k} \tag{21}$$

$$\Psi_0(z) = \frac{1}{2\pi}\sum_{k=0}^{2} e^{-2i\alpha_k}\int_{-l_k}^{l_k}\left[\frac{\overline{g_k^0(t)}}{t - z_k} - \frac{\overline{T}_k e^{i\alpha_k}}{(t - z_k)^2} g_k^0(t)\right]dt$$

where $T_k = te^{i\alpha_k} + z_k^0$, $z_k = e^{-i\alpha_k}\left(z - z_k^0\right)$, $\alpha_0 = \alpha_1 = 0$, $z_0^0 = 0$, $z_2^0 = -ih$, $l_0 = \infty$, $l_2 = \infty$.

Satisfying by functions (21) boundary conditions (20), after some transformations we get the system of three integral equations

$$\int_{-\infty}^{\infty}\left[\frac{g_0^0(t)}{t - x} + g_2^0(t)K_{0,2}(t - x) + \overline{g_2^0(t)}L_{0,2}(t - x)\right]dt = \tag{22}$$

$$= -\int_{-l_1}^{l_1}\left[g_1^0(t)K_{0,1}(t, x) + \overline{g_1^0(t)}L_{0,1}(t, x)\right]dt \qquad |x| < \infty$$

$$\int_{-\infty}^{\infty}\left[\frac{g_2^0(t)}{t-x}+g_0^0(t)K_{2,0}(t-x)+\overline{g_0^0(t)}L_{2,0}(t-x)\right]dt= \tag{23}$$

$$=-\int_{-l_1}^{l_1}\left[g_1^0(t)K_{2,1}(t,x)+\overline{g_1^0(t)}L_{2,1}(t,x)\right]dt \qquad |x|<\infty$$

$$\int_{-l_1}^{l_1}\frac{g_1^0(t)}{t-x}+\left[\int_{-l_1}^{l_1}g_1^0(t)K_{1,1}(t,x)+\overline{g_1^0(t)}L_{1,1}(t,x)\right]dt+ \tag{24}$$

$$+\int_{-\infty}^{\infty}\left[g_0^0(t)K_{1,0}(t,x)+\overline{g_0^0(t)}L_{1,0}(t,x)\right]dt+$$

$$+\int_{-\infty}^{\infty}\left[g_2^0(t)K_{1,2}(t,x)+\overline{g_2^0(t)}L_{1,2}(t,x)\right]dt=\pi f^0(x), \qquad |x|\le l_1$$

The quantities K_{nk}, L_{nk} (k,n=0,1,2) are not cited because of their bulky form. From the system of three singular integral equations (22)-(24) we exclude the two functions $g_0^0(t)$ and $g_2^0(t)$. Substituting the functions $g_0^0(x)$ and $g_2^0(x)$ found from the solution of integral equations (22) and (23), after some transformations we get one complex singular integral equation for the unknown function $g_1^0(x)$

$$\int_{-l_1}^{l_1}\frac{g_1^0(t)dt}{t-x}+\int_{-l_1}^{l_1}\left[g_1^0(t)R_{11}(t,x)+\overline{g_1^0(t)}S_{11}(t,x)\right]dt=\pi f^0(x) \qquad |x|\le l_1 \tag{25}$$

We don't cite expressions for the functions $R_{11}(t,x)$ and $S_{11}(t,x)$ because of their bulky form (they have the form similar to (V. 41) in the book [16]).

To the singular integral equation (25) for the internal crack we add the additional condition

$$\int_{-l_1}^{l_1}g_1^0(t)dt=0 \tag{26}$$

providing the uniqueness of displacements in tracing the contour of the crack in a zero approximation.

Under additional condition (26), the complex singular integral equation (25) is reduced to the system of *M* algebraic equations with respect to approximate values of the desired function $g_1^0(x_1)$ at the nodal points. For obtaining the system of algebraic equations at first in integral equation (25) and condition (26) we reduce all the integration intervals to one interval [–1, 1] by means of change of variables $t = l_1\tau$, $x = l_1\xi$ $\left(|t| < l_1,\ |x| < l_1\right)$. Look for the solution of the singular integral equation in the form

$$g_1^0(\eta) = \frac{g_1^*(\eta)}{\sqrt{1-\eta^2}} \tag{27}$$

where $g_1^*(\eta)$ is a function bounded in the interval [-1,1].

Using the quadrature formulae of Gauss type [16, 17], the singular integral equation (25) with condition (26) reduces to the system of *M* algebraic equations for defining the *M* unknowns $g_1^*(t_m)$ (*m*=1,2,…,*M*)

$$\frac{1}{M}\sum_{m=1}^{M} l_1\left[g_1^*(t_m)R_{11}(l_1t_m, l_1x_r) + \overline{g_1^*(t_m)}S_{11}(l_1t_m, l_1x_r)\right] = f^0(x_r) \tag{28}$$

$$\sum_{m=1}^{M} g_1^*(t_m) = 0, \quad t_m = \cos\frac{2m-1}{2M}\pi, \quad x_r = \cos\frac{\pi r}{M} \quad (r=1,2,\ldots,M-1)$$

For the stress intensity factors in a zero approximation, we have

$$K_{\mathrm{I}0}^{\pm} - iK_{\mathrm{II}0}^{\pm} = \mp\sqrt{\pi l_1}\, g_0^*(\pm 1) \tag{29}$$

where $g_0^*(1) = \frac{1}{M}\sum_{m=1}^{M}(-1)^m g_0^*(t_m)\cot\frac{2m-1}{4M}\pi$

$$g_0^*(-1) = \frac{1}{M}\sum_{m=1}^{M}(-1)^{M+m} g_0^*(t_m)\tan\frac{2m-1}{4M}\pi$$

In a first approximation

$$\frac{1}{M}\sum_{m=1}^{M} l_1\left[g_1^*(t_m)R(lt_m, lx_r) + \overline{g_1^*(t_m)}S(lt_m, lx_r)\right] = f_1(x_r) \tag{30}$$

$$\sum_{m=1}^{M} g_1^*(t_m) = 0\,, \qquad g_1'(\xi) = \frac{g_1^*(\xi)}{\sqrt{1-\xi^2}}$$

For the stress intensity factors in a first approximation we have

$$K_{\mathrm{I}0}^{\pm} - iK_{\mathrm{II}0}^{\pm} = \mp\sqrt{\pi d_1}\, g_1^*(\pm 1) \tag{31}$$

where $g_1^*(1) = \dfrac{1}{M}\sum_{m=1}^{M}(-1)^m g_1^*(t_m)\cot\dfrac{2m-1}{4M}\pi$

$$g_1^*(-1) = \frac{1}{M}\sum_{m=1}^{M}(-1)^{M+m} g_1^*(t_m)\tan\frac{2m-1}{4M}\pi$$

Knowing the stress intensity factors, by means of brittle fracture criterion [18, 19], for the generalized normal discontinuity

$$\cos^2\frac{\theta_*}{2}\left(K_{\mathrm{I}}\cos\frac{\theta_*}{2} - 3K_{\mathrm{II}}\sin\frac{\theta_*}{2}\right) = K_{\mathrm{I}c}\,, \qquad \theta_* = 2arctg\,\frac{K_{\mathrm{I}} \mp \sqrt{K_{\mathrm{I}}^2 + 8K_{\mathrm{II}}^2}}{4K_{\mathrm{II}}^2} \tag{32}$$

where the $K_{\mathrm{I}c}$ is a characteristic fracture toughness of the material and is determined experimentally; the sign "+" corresponds to the values of $K_{\mathrm{I}} < 0$, the sign "-" to the values of $K_{\mathrm{I}} > 0$.

Find the limit values of the external load by attaining of which the crack will be in limit-equilibrium state.

While solving algebraic systems by the Gauss method with the choice of the principal element, the number of Chebyshev nodal points was assumed to be equal to M=30.

Asphalt concrete covering of road of type 1 was accepted in place of an example of calculation. Calculations on definition of stress intensity factors were carried out. The graph of dependence of stress intensity factors on dimensionless length of the crack were represented in Figs. 2-3. Here the curve I corresponds to the smooth contour of road; curve 2 for $\delta(x) = A_1\left(\cos\dfrac{2\pi}{L_p}x - 1\right) + A_2\left(\cos\dfrac{4\pi}{L_p}x - 1\right)$, where A_1, A_2 are the amplitudes of the constituents of two-hump roughness, $x = Vt$, V is the velocity of motion in road with the components of length L_p and joining roughnesses, t is time; the curve 3 for

$\delta(x)=\sum_{n=0}^{\infty}\left(A_n\cos\frac{n\pi}{L_p}x+B\sin\frac{n\pi}{L_p}x\right)$, where A_n, B_n are non-correlated random variables satisfying the conditions $\langle A_n\rangle=0$, $\langle B_n\rangle=0$, $D\langle A_n\rangle=D\langle B_n\rangle=D_n$.

At calculations it was accepted E_1=3.2·10^3 MPa, μ_1=0.16, $\alpha_1=\pi/4$, and the crack's center at the point O_1 (0.05h; – 0.25h).

The results of calculations of stress intensity factors for the crack of opening mode (mode I) $\alpha_1=0$ from dimensionless length of the crack for different combinations of materials of covering and base are represented in Fig. 4. The road's surface is assumed to be smooth.

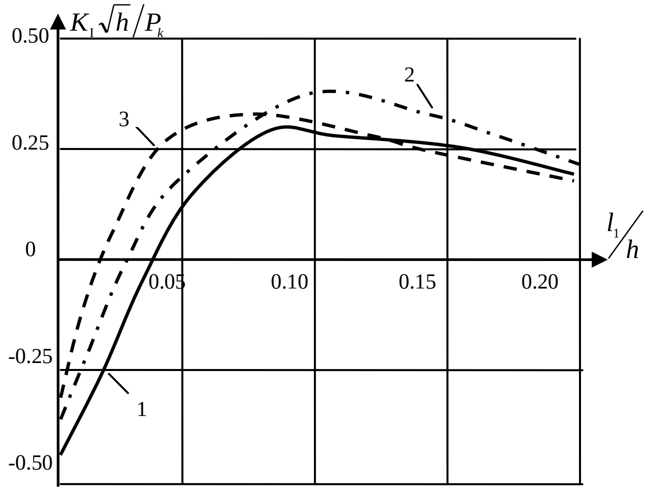

Figure 2

Dependences of the stress intensity factor K_I, on dimensionless length of the crack l_1/h

The analysis of calculations allow to make the following conclusions: a) if $G_1/G_2>1$ (G is shear modulus of the material), then for constant external load P_k and for the fixed values of other parameters of the problem, the stress intensity factor K_I increases according to increase of the crack's length. In this case there may happen fracture of the covering if the external load is such that the critical length of the crack is less than the length of the crack of the layer containing it. b) if $G_1/G_2<1$, then under constant external load and fixed values of other parameters of the problem, the dimensionless stress intensity factor $K_I/\left(P_k/\sqrt{\pi h}\right)$ at first increases according to increase of the crack's length, and then beginning with some value l_1/h, it slowly decreases.

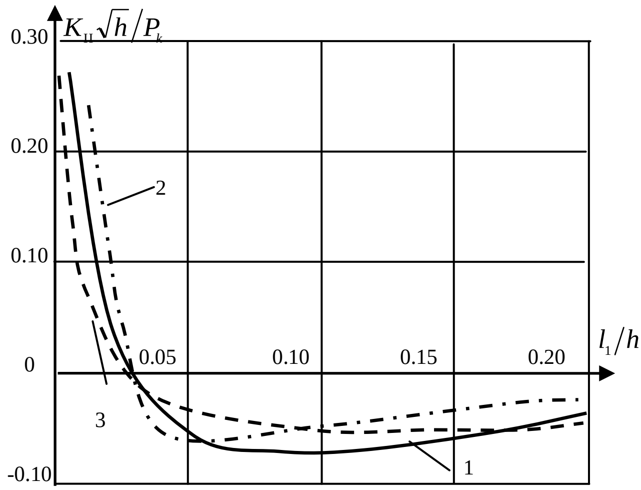

Figure 3
Dependences of the stress intensity factor K_{II}, on dimensionless length of the crack l_1/h

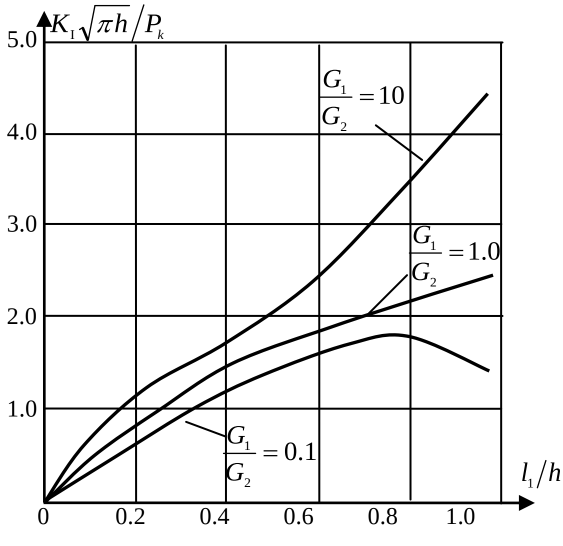

Figure 4
Dependence of the stress intensity factor K_I, on dimensionless length l_1/h of the longitudinal crack

In this case, there may happen retardation or arrest of the crack. The indicated event happens when the crack's vertex is close to the interface of media since in this case the influence of elastic base shows itself.

4 The Case of Arbitrary Number of Cracks in the Road Covering Cross Section

In the center of cracks (Fig. 1) locate the origin of local systems of coordinates $x_kO_ky_k$ whose axes x_k coincide with the lines of cracks and from angles α_k with the axis x. It is accepted that the cracks faces are free from external loads. The boundary conditions for the case under consideration are of the form (1)-(4). The stated problem is reduced to the sequence of boundary value problems in zero and first approximations.

At each approximation we use the superposition principle. We can represent the stress-strain state of a two-layer body with cracks in the form of the sum of two states. The first state will be determined from the solution of a wear contact problem on pressing out of a wheel into the road covering surface at unavailability of cracks. The second state is determined from the solution of a boundary value problem for a strip weakened by an arbitrary system of rectilinear cracks with the forces on the faces determined by the first stress state.

In a zero approximation, the boundary conditions of the second problem have the form

$$\text{for } y_k = 0 \qquad \sigma_{y_k}^{(0)} = -\sigma_k^{(0)}(x_k), \qquad \tau_{x_k y_k}^{(0)} = -\tau_k^{(0)}(x_k) \quad (k=1,2,\ldots,N) \tag{33}$$

$$\text{for } y = 0 \qquad \sigma_y^{(0)} = 0, \qquad \tau_{xy}^{(0)} = 0$$

$$\text{for } y = -h \qquad \sigma_y^{(0)} = 0, \qquad \tau_{xy}^{(0)} = 0$$

in a first approximation

$$\text{for } y_k = 0 \qquad \sigma_{y_k}^{(1)} = -\sigma_k^{(1)}(x_k), \qquad \tau_{x_k y_k}^{(1)} = -\tau_k^{(1)}(x_k) \quad (k=1,2,\ldots,N) \tag{34}$$

$$\text{for } y = 0 \qquad \sigma_y^{(1)} = 0, \qquad \tau_{xy}^{(1)} = 0$$

$$\text{for } y = -h \qquad \sigma_y^{(1)} = 0, \qquad \tau_{xy}^{(1)} = 0$$

Here $\sigma_k^{(0)}(x_k)$ and $\tau_k^{(0)}(x_k)$ are normal and tangential stresses arising in the continuous strip along the axis y_k in a zero approximation from the application of the given loads; $\sigma_k^{(1)}(x_k)$ and $\tau_k^{(1)}(x_k)$ also arise in the continuous strip along the axis y_k in a first approximation from the given loads on road covering.

The quantities $\sigma_k^{(0)}(x_k)$ and $\tau_k^{(0)}(x_k)$ and $\sigma_k^{(1)}(x_k)$, $\tau_k^{(1)}(x_k)$ are found from the relations of [13].

Consider zero approximation (33). We look for the complex potentials in the form

$$\Phi^{(0)}(z) = \frac{1}{2\pi} \sum_{k=0}^{N+1} \int_{-l_k}^{l_k} \frac{g_k^0(t)dt}{t - z_k} \tag{35}$$

$$\Psi^{(0)}(z)=\frac{1}{2\pi}\sum_{k=0}^{N+1}e^{-2i\alpha_k}\int_{-l_k}^{l_k}\left[\frac{\overline{g_k^0(t)}}{t-z_k}-\frac{\overline{T}_k e^{i\alpha_k}}{(t-z_k)^2}g_k^0(t)\right]dt \tag{36}$$

where $T_k=te^{i\alpha_k}+z_k^0,\quad z_k=e^{-i\alpha_k}\left(z-z_k^0\right).$

Having defined the stresses on the axis x_n from Kolosov-Muskheleshvili formula [14], and substituting them into boundary conditions (33), after some transformations we get the system of $N+2$ integral equations

$$\int_{-\infty}^{\infty}\left[\frac{g_0^0(t)}{t-x}+g_{N+1}^0(t)K_{0,N+1}(t-x)+\overline{g_{N+1}^0(t)}L_{0,N+1}(t-x)\right]dt= \tag{37}$$

$$=-\sum_{k=1}^{N}\int_{-l_k}^{l_k}\left[g_k^0(t)K_{0,k}(t,x)+\overline{g_k^0(t)}L_{0,k}(t,x)\right]dt \qquad |x|<\infty$$

$$\int_{-\infty}^{\infty}\left[\frac{g_{N+1}^0(t)}{t-x}+g_0^0(t)K_{N+1,0}(t-x)+\overline{g_0^0(t)}L_{N+1,0}(t-x)\right]dt= \tag{38}$$

$$=-\sum_{k=1}^{N}\int_{-l_k}^{l_k}\left[g_k^0(t)K_{N+1,k}(t,x)+\overline{g_k^0(t)}L_{N+1,k}(t,x)\right]dt \qquad |x|<\infty$$

$$\int_{-l_k}^{l_k}\frac{g_n^0(t)}{t-x}+\sum_{k\neq n}\left[\int_{-l_k}^{l_k}g_k^0(t)K_{nk}(t,x)+\overline{g_k^0(t)}L_{nk}(t,x)\right]dt+ \tag{39}$$

$$+\int_{-\infty}^{\infty}\left[g_0^0(t)K_{n,0}(t,x)+\overline{g_0^0(t)}L_{n,0}(t,x)\right]dt+$$

$$+\int_{-\infty}^{\infty}\left[g_{N+1}^0(t)K_{n,N+1}(t,x)+\overline{g_{N+1}^0(t)}L_{n,N+1}(t,x)\right]dt=\pi f_n^0(x) \qquad |x|<l_n$$

Here $K_{0,N+1}(x)=K_{N+1,0}(x)=\dfrac{x}{x^2+h^2}, \qquad L_{0,N+1}(x)=\overline{L_{N+1,0}(x)}=\dfrac{ih}{(x+ih)^2}$ (40)

$$K_{0,k}(t,x)=\frac{e^{i\alpha_k}}{2}\left(\frac{1}{T_k-x-ih/2}+\frac{1}{\overline{T}_k-x+ih/2}\right)$$

$$K_{n,0}(t,x)=\frac{1}{2}\left(\frac{1}{t+ih/2-X_n}+\frac{e^{-2i\alpha_n}}{t-ih/2-\overline{X}_n}\right)$$

$$L_{0,k}(t,x)=\frac{e^{-i\alpha_k}}{2}\frac{\left(\overline{T}_k-T_k+ih\right)}{\left(\overline{T}_k-x+ih/2\right)^2}, \qquad L_{N+1,k}(t,x)=\frac{e^{-i\alpha_k}}{2}\frac{\left(\overline{T}_k-T_k-ih\right)}{\left(\overline{T}_k-x-ih/2\right)^2}$$

$$L_{n,0}(t,x)=\frac{1}{2}\left(\frac{1}{t-ih/2-\overline{X}_n}-\frac{t+ih/2-X_n}{\left(t-ih/2-\overline{X}_n\right)^2}e^{-2i\alpha_n}\right)$$

$$L_{n,N+1}(t,x)=\frac{1}{2}\left(\frac{1}{t+ih/2-\overline{X}_n}-\frac{t-ih/2-X_n}{\left(t+ih/2-\overline{X}_n\right)^2}e^{-2i\alpha_n}\right)$$

$$K_{N+1,k}(t,x)=\frac{e^{i\alpha_k}}{2}\left(\frac{1}{T_k-x+ih/2}+\frac{1}{\overline{T}_k-x-ih/2}\right)$$

$$K_{n,N+1}(t,x)=\frac{1}{2}\left(\frac{1}{t-ih/2-X_n}+\frac{e^{-2i\alpha_n}}{t+ih/2-\overline{X}_n}\right)$$

$$K_{nk}(t,x)=\frac{e^{i\alpha_k}}{2}\left(\frac{1}{T_k-X_n}+\frac{e^{-2i\alpha_n}}{\left(\overline{T}_k-\overline{X}_n\right)^2}\right)$$

$$L_{nk}(t,x)=\frac{e^{-i\alpha_k}}{2}\left(\frac{1}{\overline{T}_k-\overline{X}_n}-\frac{T_k-X_n}{\left(\overline{T}_k-\overline{X}_n\right)^2}e^{-2i\alpha_n}\right), \qquad X_n=xe^{i\alpha_n}+z_n^0$$

For convenience, in (37), (38), (39) and in what follows the index x_n is omitted. From the system of N+2 singular integral equations we exclude the two unknown functions $g_0^0(t)$ and $g_{N+1}^0(t)$.

After some transformations we get a system N singular integral equations of the problem under consideration in a zero approximation

$$\int_{-l_k}^{l_k}\frac{g_k^0(t)\,dt}{t-x}+\sum_{k=1}^{N}\int_{-l_k}^{l_k}\left[g_k^0(t)R_{nk}(t,x)+\overline{g_k^0(t)}S_{nk}(t,x)\right]dt=\pi f_n^0(x_n) \quad |x|\le l_n \tag{41}$$

(n=1,2,…,N)

$$R_{nk}(t,x)=(1-\delta_{nk})K_{nk}(t,x)+r_{nk}(t,x), \qquad S_{nk}(t,x)=(1-\delta_{nk})L_{nk}(t,x)+s_{nk}(t,x) \tag{42}$$

$$r_{nk}(t,x)=\int_{-\infty}^{\infty}\Big[K_{n,0}(\tau,x)M_{0,k}(t,\tau)+L_{n,0}(\tau,x)\overline{N_{0,k}(t,\tau)}+ \tag{43}$$

$$+K_{n,N+1}(\tau,x)M_{N+1,k}(t,\tau)+L_{n,N+1}(\tau,x)\overline{N_{N+1,k}(t,\tau)}\Big]\,d\tau$$

$$s_{nk}(t,x)=\int_{-\infty}^{\infty}\Big[K_{n,0}(\tau,x)N_{0,k}(t,\tau)+L_{n,0}(\tau,x)\overline{M_{0,k}(t,\tau)}+$$

$$+K_{n,N+1}(\tau,x)N_{N+1,k}(t,\tau)+L_{n,N+1}(\tau,x)\overline{M_{N+1,k}(t,\tau)}\Big]\,d\tau$$

After substituting functions $M_{0,k}(u,x)$, $N_{0,k}(u,x)$, $M_{N+1,k}(u,x)$, $N_{N+1,k}(u,x)$ into (43), the kernels $r_{nk}(t,x)$ and $s_{nk}(t,x)$ will be represented by triple integrals. After integration these expressions may be represented by one-fold integrals.

Omitting very bulky calculations, finally for the kernels $r_{nk}(t,x)$ and $s_{nk}(t,x)$ we find

$$r_{nk}(t,x)=\int_{0}^{\infty}\left[\left(\frac{1}{shhs+hs}+\frac{1}{shhs-hs}\right)H_{nk}\left(X_n,T_k,s,\alpha_n,\alpha_k\right)+\right. \tag{44}$$

$$\left.+\left(\frac{1}{shhs+hs}-\frac{1}{shhs-hs}\right)G_{nk}\left(X_n,T_k,s,\alpha_n,\alpha_k\right)\right]ds$$

$$s_{nk}(t,x)=\int_{0}^{\infty}\left[\left(\frac{1}{shhs+hs}-\frac{1}{shhs-hs}\right)H_{nk}\left(X_n,\overline{T}_k,s,\alpha_n,-\alpha_k\right)+\right.$$

$$\left.+\left(\frac{1}{shhs+hs}+\frac{1}{shhs-hs}\right)G_{nk}\left(X_n,\overline{T}_k,s,\alpha_n,-\alpha_k\right)\right]ds$$

Here $$H_{nk}\left(X_n,\overline{T}_k,s,\alpha_n,-\alpha_k\right)=\frac{e^{i\alpha_k}}{4}\Big\{\sin\left(X_n-\overline{T}_k\right)s-\sin\left(T_k-\overline{X}_n\right)s\Big\langle hs+ \tag{45}$$

$$+e^{-2i\alpha_n}\left[1-hs+s^2\left(T_k-\overline{T}_k\right)\left(\overline{X}_n-X_n\right)+h^2s^2\right]\Big\rangle+\Big\langle s\left(T_k-\overline{T}_k\right)-$$

$$-e^{-2i\alpha_n}\left[\left(T_k-\overline{T}_k\right)+hs^2\left(\overline{X}_n-X_n-T_k+\overline{T}_k\right)\right]\Big\rangle\cos\left(T_k-\overline{X}_n\right)s+$$

$$+e^{-hs}\left[\sin\left(T_k-X_n\right)s+e^{-2i\alpha_n}\sin\left(\overline{T}_k-\overline{X}_n\right)s\right]\Big\};$$

$$G_{nk}\left(X_n,T_k,s,\alpha_n,\alpha_k\right)=\frac{e^{i\alpha_k}}{4}\Big\{-\left[1+e^{-2i\alpha_n}\left(-1+hs\right)\right]\sin\left(\overline{T}_k-\overline{X}_n\right)s-$$

$$-hs\sin\left(T_k-X_n\right)s-s\left(T_k-\overline{T}_k\right)\cos\left(T_k-X_n\right)s-$$

$$-e^{-2i\alpha_n}\left(\overline{X}_n-X_n\right)s\cos\left(\overline{T}_k-\overline{X}_n\right)s+e^{-hs}\Big[\sin\left(T_k-\overline{X}_n\right)s-$$

$$-e^{-2i\alpha_n}\sin\left(T_k-\overline{X}_n\right)s+e^{-2i\alpha_n}\sin\left(\overline{X}_n-X_n-T_k+\overline{T}_k\right)s\cos\left(T_k-\overline{X}_n\right)s\Big]\Big\}$$

Note that the functions $r_{nk}(t,x)$ and $s_{nk}(t,x)$ are regular. They determine the influence of the boundaries of the strip on stress state near the cracks vertices. To the system of singular integral equations (41) for the internal cracks we should add the following additional conditions

$$\int_{-l_k}^{l_k} g_k^0(t)dt = 0 \qquad (k = 1, 2, \ldots, N) \tag{46}$$

Using the procedure for converting a system to an algebraic [16, 17], the system of singular integral equations (41) with conditions (46) is reduced to a system of $N\times M$ algebraic equations for determining the $N\times M$ unknowns $g_k^0(t_m)$:

$$\frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{N} l_k\left[g_k^0(t_m)R_{nk}\left(l_k t_m, l_n x_r\right) + \overline{g_k^0(t_m)}\, S_{nk}\left(l_k t_m, l_n x_r\right)\right] = f_n^0(x_r^0) \tag{47}$$

$$\sum_{m=1}^{M} g_n^0(t_m) = 0 \qquad (n,k=1,2,\ldots,N;\ m=1,2,\ldots,M)$$

If in (47) we pass to complexly conjugate values, we get one more $N\times M$ algebraic equations.

For the stress intensity factors in the vicinity of the cracks tips, in a zero approximation we find:

at the right vertex of the crack

$$K_{In}^{(0)} - iK_{IIn}^{(0)} = \sqrt{\pi l_n}\,\frac{1}{M}\sum_{m=1}^{M}(-1)^m g_n^0(t_m)\cot\frac{2m-1}{4M}\pi \tag{48}$$

at the left vertex of the crack

$$K_{In}^{(0)} - iK_{IIn}^{(0)} = \sqrt{\pi l_n}\,\frac{1}{M}\sum_{m=1}^{M}(-1)^{M+m} g_n^0(t_m)\tan\frac{2m-1}{4M}\pi \tag{49}$$

Now consider the solution of problem (34) in a first approximation.

Behaving as above, we get a system of singular integral equations of a first approximation

$$\int_{-l_k}^{l_k}\frac{g_k^1(t)dt}{t-x} + \sum_{k=1}^{N}\int_{-l_k}^{l_k}\left[g_k^1(t)R_{nk}(t,x) + \overline{g_k^1(t)}S_{nk}(t,x)\right]dt = \pi f_n^1(x_n) \qquad |x| \le l_n \tag{50}$$

To the system of singular integral equations (50) we should add the additional equalities

$$\int_{-l_k}^{l_k} g_k^1(t)dt = 0 \qquad (k = 1, 2, \ldots, N) \tag{51}$$

As in a zero approximation, the system of complex singular equations (50) with conditions (51) by means of the algebraization procedure [16, 17] is reduced to the system of $N\times M$ algebraic equations with respect to $N\times M$ unknown $g_k^1(t_m)$ (k=1,2,…,N; m=1,2,…,M):

$$\frac{1}{M}\sum_{m=1}^{M}\sum_{k=1}^{N} l_k\left[g_k^1(t_m)R_{nk}\left(l_k t_m, l_n x_r\right)+\overline{g_k^1(t_m)}\,S_{nk}\left(l_k t_m, l_n x_r\right)\right]= f_n^1(x_r^0) \tag{52}$$

$$\left(r=1,2,\ldots,M-1\right)$$

$$\sum_{m=1}^{M} g_n^1(t_m)=0$$

If in (52) we pass to complexly conjugate values, we get one more $N\times M$ algebraic equations.

For the stress intensity factors in the vicinity of the cracks tips in a first approximation we get

at the right vertex of the crack

$$K_{\mathrm{I}n}^{(1)}-iK_{\mathrm{II}n}^{(1)}=\sqrt{\pi l_n}\,\frac{1}{M}\sum_{m=1}^{M}(-1)^m g_n^1(t_m)\cot\frac{2m-1}{4M}\pi \tag{53}$$

at the left vertex of the crack

$$K_{\mathrm{I}n}^{(1)}-iK_{\mathrm{II}n}^{(1)}=\sqrt{\pi l_n}\,\frac{1}{M}\sum_{m=1}^{M}(-1)^{M+m} g_n^1(t_m)\tan\frac{2m-1}{4M}\pi \tag{54}$$

Finally, for the stress intensity factors we have

$$K_{\mathrm{I}n}=K_{\mathrm{I}n}^{(0)}+\varepsilon K_{\mathrm{I}n}^{(1)}+\ldots,\qquad K_{\mathrm{II}n}=K_{\mathrm{II}n}^{(0)}+\varepsilon K_{\mathrm{II}n}^{(1)}+\ldots$$

Conclusions

Experimental data of practice of exploitation of the pair "road covering-elastic base" convincingly shows that by designing new constructions of motor roads it is necessary to take into attention the cases when in road covering there may arise cracks. The existing methods of strength analysis of road coverings ignore this circumstance. Such a situation makes impossible to design road coverings of minimal specific consumption of materials under guaranteed reliability and durability. In this connection, it is necessary to realize the limiting analysis of the pair "road covering-base" in order to establish that the presupposed initial cracks located unfavorably will not grow to catastrophic sizes and will not cause fracture during the rated service life. The size of the initial minimal crack should be considered as a design characteristics of the covering material.

Based around the suggested design model taking into account crack-like defects in road covering, a method for calculating the fracture parameters of road covering with regard to real roughness surface of the road is developed. The elaborated calculation method, by means of definition of stress intensity factors, allows to predict the growth of available cracks in road covering, to take into account not only separately each realization of the roughness profile (deterministic approach) and also to carry out statistic description of surface roughnesses of the road by realization of a random function, to evaluate different factors (constructive, technological, operational) for road covering strength.

Numerical realization of the obtained dependences allows to solve the following practically important problems:

1) definition of critical sizes of a crack under known loads, stresses and fracture toughness. These informations may be used by developing requirements to decision abilities of the used methods of defectoscopy.

2) definition of critical level of stresses depending on external loads and parameters of brittle strength.

These informations may be used by developing technological processes for lowering the level of residual stresses.

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