

# Fast Fourier Transform in Papermaking and Printing: Two Application Examples

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*Abstract: The Fast Fourier transform was applied to demonstrate how to solve two related technological problems in the fields of papermaking and printing using ImageJ, a public domain Java image processing program. By converting a digital image of a multifunctional office paper surface from the spatial- into the frequency domain followed by an appropriate filtering, it was possible to identify and separate two different types of patterns – non-periodic structures such as formation, and periodic structures such as fabric marks. In addition, the moiré pattern, a negative phenomenon often occurring in printing, was eliminated to a large extent. The method proved to be a valuable tool in investigating and quantifying various paper and print quality-related phenomena.*

*Keywords: image processing; Fast Fourier transform; fabric marks; paper formation; print quality; moiré*

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## 1 Introduction

With the rapid increase in computer processing power and data storage capacities, accompanied by a constant decrease in hardware and software prices, complex scientific and engineering problems can now be solved in a very short time period using a typical personal computer. Fourier transform is a mathematical tool frequently used in a number of technical fields, as diverse as applied mechanics, biomedical engineering [2], image- and sound compression, NMR and MR imaging and partial differential equation solving [3].

In our study, a Fast Fourier transform was implemented as an advanced digital image processing operation used for analyzing two related technical applications:

first, the separation of two types of patterns that are generated during an industrial paper making process – paper formation and fabric (cloth) marks – and second, the moiré phenomenon, which often occurs in printing. The focus of this article is on demonstrating the applicability of FFT in two real-world situations rather than on going into details of its technical implementation.

## 2 Fourier Transform

According to Fourier's theorem, any continuous periodic signal (function) can be represented as the sum of properly chosen sinusoidal waves, i.e. by a series of sine and cosine terms with appropriate frequency, amplitude, and phase [15]:

$$g(x) = \sum_{k=0}^{\infty} [A_k \cos(k\omega_0 x) + B_k \sin(k\omega_0 x)] \quad (1)$$

where the constants  $A_k$  and  $B_k$  denote the Fourier coefficients of the function  $g(x)$ , from which they can be derived in a process referred to as Fourier analysis [4]. For non-periodic functions, a modified version of Eq. (1) – a Fourier integral – applies:

$$g(x) = \int_0^{\infty} A_{\omega} \cos(\omega x) + B_{\omega} \sin(\omega x) d\omega \quad (2)$$

where  $A_{\omega}$  and  $B_{\omega}$  denote the weights for the corresponding cosine and sine functions with the continuous frequency  $\omega$ . The weights can be computed as follows:

$$A_{\omega} = A(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} g(x) \cdot \cos(\omega x) dx \quad (3)$$

$$B_{\omega} = B(\omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} g(x) \cdot \sin(\omega x) dx \quad (4)$$

The original signal  $g(x)$  is uniquely represented by the correspondent coefficient functions  $A(\omega)$  and  $B(\omega)$ , which hold the continuous distribution – a spectrum – of frequency components contained in the original signal. In the Fourier transform (FT), both the original signal and its spectrum are treated as complex-valued functions yielding the following well-known expression for the continuous Fourier spectrum  $G(\omega)$ :

$$G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) \cdot [\cos(\omega x) - i \cdot \sin(\omega x)] dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(x) \cdot e^{-i\omega x} dx \quad (5)$$

While the (forward) FT shown in Eq. (5) enables the computation of  $G(\omega)$  from  $g(x)$ , the inverse (backward) FT makes it possible to reconstruct  $g(x)$  from  $G(\omega)$ :

$$g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) \cdot [\cos(\omega x) + i \cdot \sin(\omega x)] d\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) \cdot e^{i\omega x} d\omega \quad (6)$$

The original function "signal space" (domain) and its "frequency space" representation – the spectrum produced by the FT – are two alternative and interchangeable mathematical representations of the same function.

Since digital computers can only process discrete signals, a continuous function should first be converted into a discrete one through sampling. Then the discrete FT (DFT) can be used with the forward transform defined for a discrete signal  $g(u)$  of length  $M$  ( $u = 0 \dots M-1$ ) as:

$$\begin{aligned} G(m) &= \frac{1}{\sqrt{M}} \sum_{u=0}^{M-1} g(u) \cdot \left[ \cos\left(2\pi \frac{mu}{M}\right) - i \cdot \sin\left(2\pi \frac{mu}{M}\right) \right] \\ &= \frac{1}{\sqrt{M}} \sum_{u=0}^{M-1} g(u) \cdot e^{-i2\pi(mu/M)} \quad \text{for } 0 \leq m < M \end{aligned} \quad (7)$$

and the inverse transform (DFT<sup>-1</sup>) as:

$$\begin{aligned} g(u) &= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} G(m) \cdot \left[ \cos\left(2\pi \frac{mu}{M}\right) + i \cdot \sin\left(2\pi \frac{mu}{M}\right) \right] \\ &= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} G(m) \cdot e^{i2\pi(mu/M)} \quad \text{for } 0 \leq u < M \end{aligned} \quad (8)$$

The Fast Fourier transform (FFT) is one of the most efficient algorithms for implementing, i.e. computing, the DFT and its inverse. Its main advantage over other approaches lies in its speed, since the required computation time can be reduced by several orders of magnitude. The mathematical details of this complex procedure can be found elsewhere; see, e.g. [13] or the original article by Cooley and Tukey [6].

When DFT is applied to a digital image, its spatial information is expressed as frequency and phase data. The frequency component can be displayed via the power spectrum, as shown in the Experimental part.

Various types of filtering, such as low-, high- or band-pass, can be performed on the Fourier transformed image via its multiplication by the appropriate filter function. This operation is identical to convolution in the spatial domain, but is often faster and more suitable for some applications.

### 3 Paper Formation and Periodic Marks

The modern industrial paper manufacturing process on a paper or board machine consists of a number of consecutive technological stages, such as sheet forming, dewatering, wet pressing, drying, roll calendering, coating, and others. Each of these operations has a distinctive and profound effect on the final paper sheet bulk and surface properties. For instance, when a soft, wet paper passes a nip between a pair of press section rolls designed to remove water mechanically and to smooth and compress the sheet, it is exposed to the pressures ranging from 400-8500 N/cm [5].

At various paper machine locations – the forming screen, wet press and drying section – a sheet of paper comes into a direct contact with synthetic cloth (fabric, felt). Depending on the paper grade and exerted pressure, the surface structure – e.g. a weaving pattern – of the fabric material can leave more or less noticeable periodic, regular markings in the paper surface and consequently can adversely affect its appearance. In addition, due to the quasi-random process of the cellulose fibres' distribution, which takes place during the paper forming stage, the paper sheet is also inevitably characterized by having a non-uniform, irregular structure of fibres and other paper ingredients, resulting in a cloudy visual appearance, known as (uneven) formation or cloudiness. Either or both of these two phenomena, if too pronounced, can severely degrade the end product quality.

In the past, the detection, separation and quantification of these two types of structures in paper proved to be a difficult task. Easy access to PC-based image processing and analysis systems, however, has made it possible to successfully investigate these and other issues related to paper science and technology. Our approach is based on the work done by dr. I'Anson from UMIST [8], [9] who had already implemented FFT in the 1990s to solve numerous real-world, paper quality related problems.

### 4 Moiré in Printing

The moiré effect is a visual perception that occurs when viewing a set of lines or dots that is superimposed on another set of lines or dots, where the sets differ in relative size, angle, or spacing. The moiré phenomenon has been known for a long time. It was already used by the Chinese in ancient times for creating an effect of dynamic patterns in silk cloth. However, modern scientific research into the moiré phenomenon and its application started only in the second half of the 19th century. The first significant steps in the introduction of the Fourier theory to the study of the moiré phenomenon can be traced back to the 1960s and 1970s [1].

The moiré possesses an enormous magnification power – in some cases a million times or more – and provides an extremely sensitive means of visually detecting minute differences in almost identical repeating figures. Because of this property, the moiré phenomenon has useful applications in areas such as crystallography to check crystal defects, for measuring refractive index, in electrophoresis, for testing diffraction gratings and lenses [10], [12], [17], [18]. The only requirement for a moiré pattern is that the interacting figures with two or more periodic structures and with some sort of solid and open regions are superimposed. The solid regions can have different shapes, such as straight, curved or wiggly lines, or dots, ellipses etc. This may happen in two ways; if two periodic structures have close but different periods superimposed in parallel or if two structures have an identical period intersected at a small angle. The result of the periodic interaction is that a visible regular pattern is clearly observed at the intersection, although it does not appear in the original structures.

A moiré can produce interesting and beautiful geometric patterns, but it always degrades the quality and resolution of graphic images. It is the most noticeable spatial domain problem in color printing. Two possible reasons for a moiré pattern appearing in printing are:

- The superposition of halftone screens of different primary colors (C, M, Y and K).
- If the original image is already half-toned or contains some other periodic fine details, an unwanted pattern may appear as an interference between the original screen ruling and the printed halftone frequency.

The moiré effect can be observed in one-color printing, but it is more problematic in color printing, where color line screens are needed. This kind of interaction not only gives texture differences but also color differences [7]. The knowledge of understanding the natural causes of moiré patterns will make it possible to avoid or minimize their adverse effects on color printing. Literature cites three approaches to the moiré treatment: a simple qualitative measure, a geometric model and a complete Fourier spectral analysis [11].

## 5 Experimental

In our study FFT was applied using ImageJ [14], a public domain Java image processing program developed by Wayne Rasband at the U.S. National Institutes of Health (NIH). To separate paper formation from its periodic marks, the following consecutive image processing operations were performed:

### 1. Image acquisition and preprocessing

Two 80 g/m<sup>2</sup> multifunctional office papers Navigator Universal – No. 1 (see Figure 1) and No. 2 (see Figure 2) – were scanned using a Hewlett-Packard

ScanJet G4010 flat-bed scanner in transmission mode at 200 dpi, and the acquired RGB color images (65 x 65 mm = 512 x 512 pxl) were converted to 8-bit grayscale images and saved as uncompressed TIFF files. The contrast of the images was improved by performing an automatic contrast adjustment ("histogram stretching") operation so that the available range of 256 values was fully covered.

2. Transforming grayscale images from spatial to frequency domain using FFT and displaying its power spectrum

Power spectrum displays squared amplitudes of the frequency spectrum in a 2D map (see e.g. Figure 1c and Figure 2b) having a distinct pattern, typically symmetrical about the origin [16]. Here lower frequencies lie closer to the origin and higher ones closer to the edges of the image.

3. Creating and applying appropriate filters (masks) using information from the power spectrum

Filters that were either transparent (white; pixel value = 255) or nontransparent (black; pixel value = 0) in the appropriate frequency range were designed and subsequently applied to passing through either exclusively regular (high frequencies) or solely irregular (low frequencies) image structures. After performing the inverse DFT, only selected frequencies appeared in the image.

The moiré removal principle was demonstrated using a color printed sample produced with a four-color offset printing machine, a Heidelberg SM 52 (screen frequency 70 lines/cm). A moiré pattern occurred because the order of the four printing forms was deliberately altered. The color cyan was printed with a printing plate prepared on a yellow color separation. Magenta was printed with a printing plate for cyan and yellow with a printing plate for magenta color. The printed sample (paper Biogloss 135 g/m<sup>2</sup>, A3 format) was scanned with Hewlett-Packard Photosmart C6280 in the normal reflection mode with a resolution of 150 dpi, and the resulting image was saved in an uncompressed TIFF format. The image was further processed as discussed below.

## 6 Results and Discussion

### 6.1 Separation of Paper Formation and Periodic Marks

The results of image processing operations are depicted in Figure 1. Figure 1a shows the original grayscale image of the paper sample No. 1, (b) is its contrast-enhanced version to increase the visibility of regular structures, while (c) shows the enlarged central part (200 x 200 pxl) of the power spectrum image having

several sharp peaks positioned in a regular pattern around the central white blob. Note that the brightness was reduced to enable better visualisation of the peaks. By drawing two vertically placed white circles ( $2r = 4$  pxl) using the ImageJ drawing tool at the appropriate locations on the power spectrum image (d) and transforming the frequency domain image back to the spatial domain, horizontal periodic marks become visible (g). In a similar manner, the horizontal white circles (e) produce an image with distinctive vertical stripes (h). The combined effect of a four-circles' filter (f) on visualizing both the horizontal and vertical marks is shown in (i). Note that in the output images (see, e.g. (g), (h), (i)) apart from this high-frequency image information corresponding to the periodic, regular structures, such as wire and/or felt marks, also a substantial amount of non-periodic, irregular structures stemming mainly from paper formation is present.

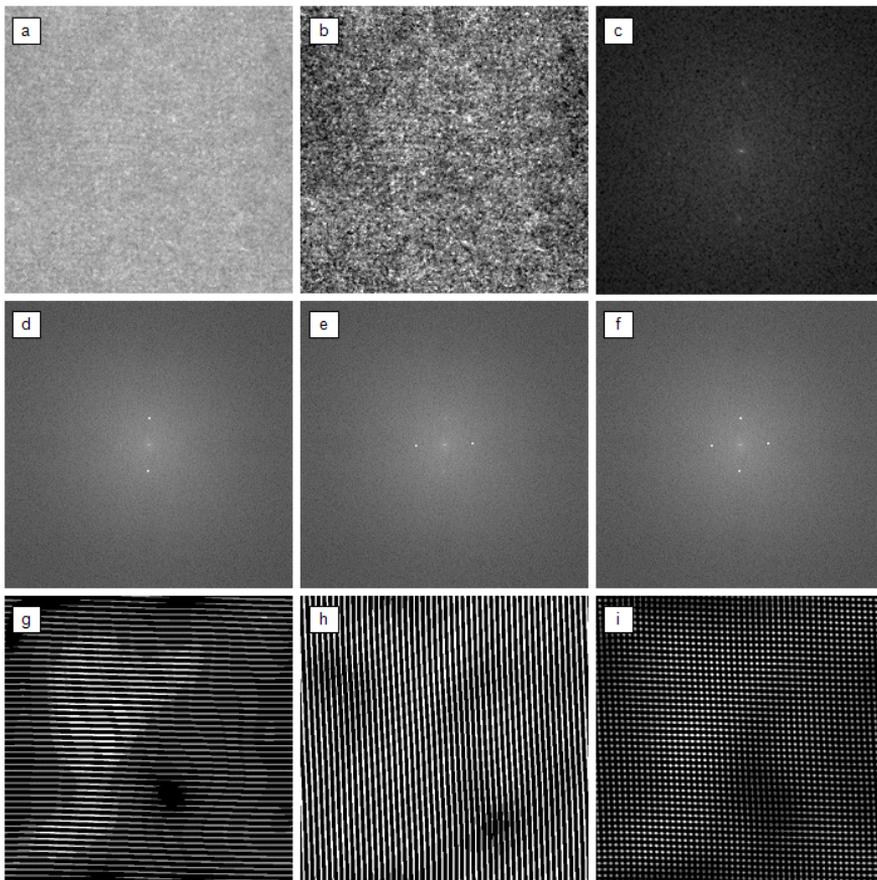


Figure 1

Results of implementing FFT and subsequent filtering on office paper No. 1. See text for details.

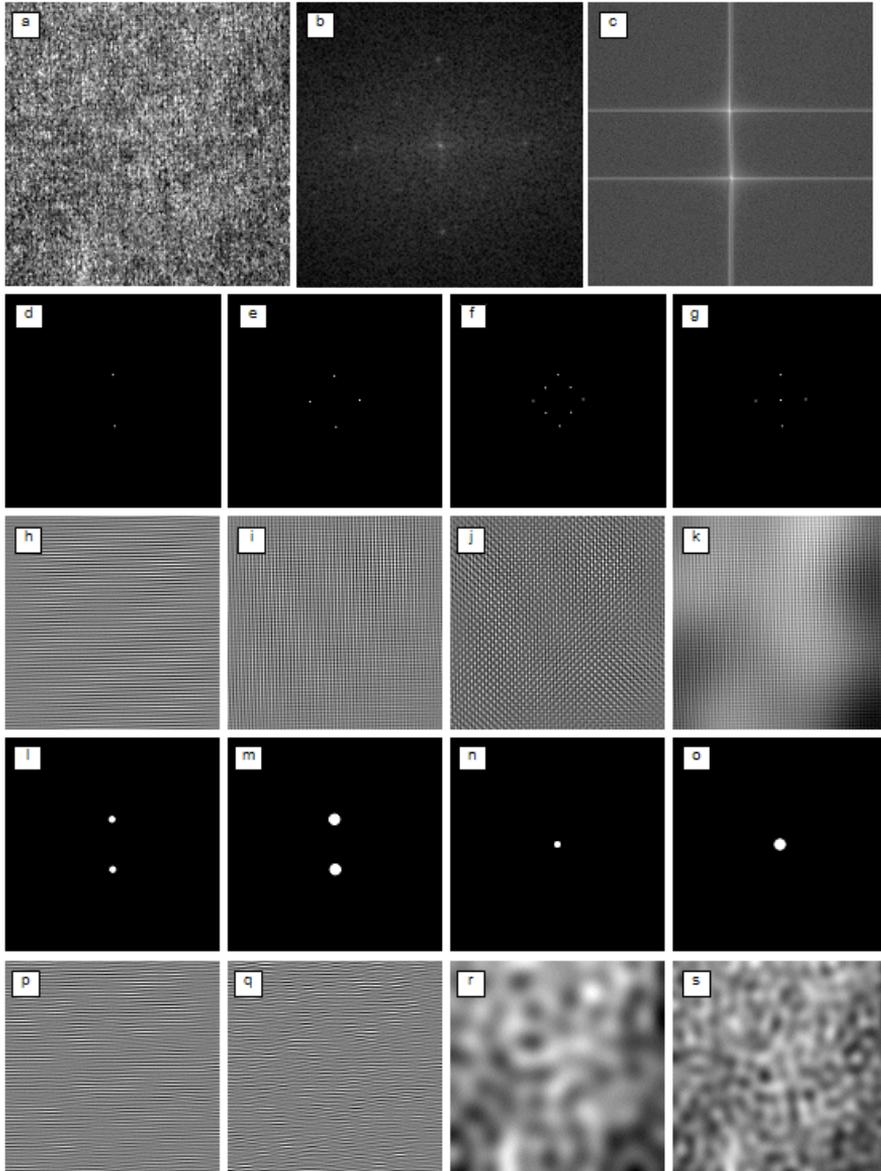


Figure 2

Results of implementing FFT and subsequent filtering on office paper No. 2. See text for details.

Alternatively, one can design circular filters from scratch using the amplitude and frequency information from the power spectrum and perform filtering of the image using these ideal low- or high-pass filters. Black filter pixels block corresponding spatial frequencies while white pixels cause respective frequencies

to pass without attenuation (see also discussion on the moiré pattern removal, below). The filter should be symmetric with respect to the center, so that no artefacts are generated.

Figure 2 presents the results of adopting this approach. The paper sample No. 2 was, after scanning and performing contrast adjustment, (a) FFT transformed – (b) again displays only the enlarged central part of the power spectrum image – and filtered. The effects of applying filters consisting of white circles that varied in position, number and size are shown. Filtering using vertical and horizontal circles (d), (e) results in emphasizing horizontal and vertical lines (h), (i), respectively, as already seen in Figure 1. Since due to the black background of the filter masks no other frequencies are allowed to pass through, results here are much better, i.e., the paper formation-related structures have been almost entirely eliminated. Note that applying FFT on one of the resulting images – e.g. (h) – reveals the exact location of the corresponding white filter circles – as shown in (c). An increase in the size of the two vertical circles – from 4 (d) to 16 (l) to 28 (m) pixels – has a consequence of producing images with progressively less articulated horizontal stripes – see (h), (p) and (q). A closer inspection of the image power spectrum (b) reveals four additional high-frequency structures, so the corresponding symmetrical filter mask with eight circles (f), when applied to the image, leads to (j), displaying another two diagonal (top-left to bottom right and top-right to bottom-left) regular marks. In practice, information about the directionality, i.e. the angle, periodicity and other characteristics of these structures, can be very valuable to both the paper maker and the fabric manufacturer when examining problems that might occur during the paper or wire/felt production. In particular, the geometrical arrangement of peaks in the power spectrum image, such as Figure 1c or Figure 2b, can serve as a kind of fingerprint that is characteristic for each paper machine-paper grade combination; thus a paper mill operator can, especially when having access to an actual piece of cloth from various paper machine sections, unambiguously determine whether the paper marks are caused by a forming wire, a press felt cloth or a dryer fabric [8].

If even a very small number of low spatial frequencies corresponding to the central white blob are allowed to pass through (g), a clearly visible irregular pattern typical of paper formation appears (k). By increasing the central white circle size from 4 (g) to 16 (n) and 28 pixels (o), a progressively more detailed structure of formation becomes evident – see (k), (r), (s).

## 6.2 Moiré Pattern Removal

First the moiré-containing grayscale image (Figure 3a) was opened in ImageJ. Then a part of the image where only the moiré pattern – without any other disturbing image details, i.e. irregular structures – was selected (b), isolated and saved as a TIFF file. An ImageJ FFT plugin converted this image from a spatial into a frequency domain, generating its power spectrum image (c).

Similarly to the above discussed problem of separating the paper formation from the regular markings, the power – FFT – spectrum of the moiré-containing image also shows several white spots (Figure 3c), producing, however, a considerably more complex pattern. Again, the exact location of the white peaks is the key that enables one to determine what sort of variation is present in the original image. White dots in the centre of the power spectrum image correspond to non-periodic structures, while white regions located towards the edges of the FFT image are related to periodic structures.

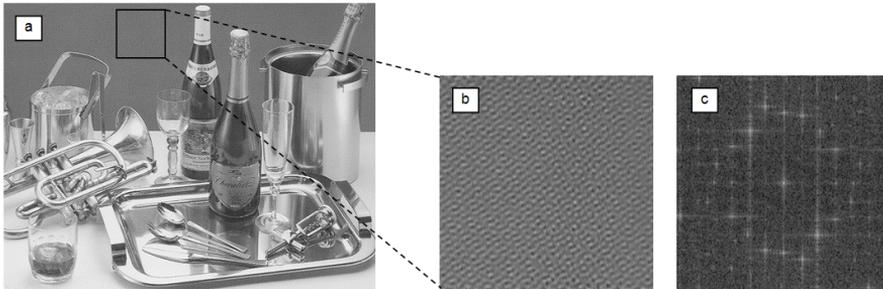


Figure 3

Original image (a), part of the image background with moiré pattern (b) and its power spectrum (c).

Using this information, a bilevel filter (mask) shown in Figure 4a and its inverted version, in Figure 4c, were designed. Note that both are characterized by having either a white or a black central circle. As was already seen in the previous example, due to this circle, the moiré pattern could be neither completely isolated and displayed (b) nor removed (d) in the resulting spatial domain images. In the case of (b), apart from the high frequency (moiré) structures, some low frequency information, corresponding to the picture scene, was also allowed to pass through. In the case of (d), on the other hand, not only were the majority of high frequencies suppressed by the black filter regions (crosses and dots), but also the image details themselves were somewhat blocked by the central black spot, resulting in a washed-out, low contrast image.

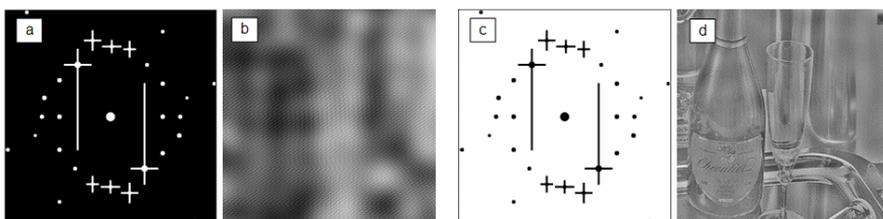


Figure 4

High frequency-passing (a) and -blocking (c) filters with a central circle and the resulting spatial domain images (b, d)

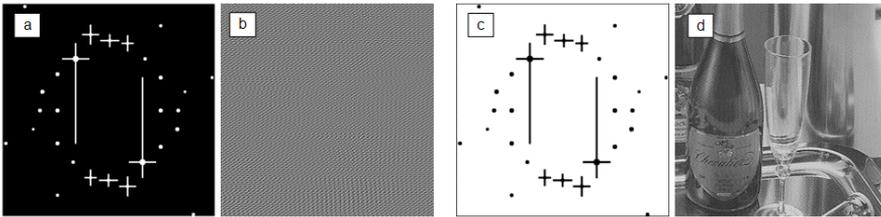


Figure 5

High frequency-passing (a) and -blocking (c) filters without a central circle and the resulting spatial domain images (b, d)

On the other hand, when the created filters did not contain the central circular masking regions, see Figures 5a and 5c, the results of applying these images to the original one (Figure 3a) were much better. In the case of using the filter that preserves exclusively high-frequency information (a), only the moiré pattern was visible (b), while when adopting the filter that attenuates or stops all high-frequency features in the original image (c), the result was a clear, almost moiré-free picture (d).

## Conclusions

As the two presented examples demonstrate, FFT is a useful technique for transforming images into the frequency domain where appropriate either low-pass or high-pass filtering can be carried out. This enables on to make changes to the original images that would be very difficult to perform in the spatial domain using convolution operations. The clear separation of the two distinctive types of patterns present in a paper or board – non-periodic structures such as formation and periodic structures such as fabric marks – and a more detailed characterization of the latter, such as identification of the marks' angle, periodicity and origin, can be made using a standard PC with open-source image processing software.

A moiré pattern, a negative phenomenon often occurring in printing, was also successfully eliminated using FFT. Conventional, spatial filtering-based methods for dealing with moiré – e.g. the Gaussian blur – act on the image as a whole, resulting in a decrease in sharpness, and they are in general also less successful in removing a moiré pattern.

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