Formulation of predictive model for the compressive strength of oyster shell powder-cement concrete using Scheffe's simplex lattice theory

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Abstract
This empirical study was carried out to formulate and validate a predictive model for the compressive strength of oyster shell powder-cement concrete using Scheffe's simplex lattice theory, so as to ensure economic usage of readily available oyster shells. A total of 90 cubes of concrete were cast to formulate and validate the model for the compressive strength of oyster shell powder (OSP)-cement concrete using Scheffe's (5, 2) simplex lattice theory. The formulated model was tested for adequacy using the Student t-test. It was observed that the model results agree with those of the experiments. Hence, the model is adequate and can predict the compressive strength, given the mix proportions. The model gave highest compressive strength of 30.81 N/mm^2 corresponding to mix ratio of 0.54:0.815:2.045:3.925:0.185 for water, cement, sand, granite and OSP respectively. The developed model also gave minimum compressive strength of 17.85 N/mm^2 corresponding to mix ratio of 0.525:0.825:2.2:4.05:0.175 for water, cement, sand, granite and OSP respectively. With this formulated model, any point on the simplex can easily be derived.

Key words: concrete, oyster shells, compressive strength, Scheffe, simplex
Kulcsszavak: beton, osztriga héj, nyomószilárdság, Scheffe, szimplex

1. Introduction

When a product is formed by mixing together two or more ingredients, the product is called a mixture and the ingredients are called mixture components [1]. In the case of concrete (mixture) and mixture components (cement, sand, granite, water and admixture or supplementary cementitious material), there is need to develop a way of optimally combining these ingredients, with a view to economizing our scarce resources, without compromise on the rheological and hardened properties of concrete produced. According to [2], it is important to find the optimum dosage and substitution ratio, because application of supplementary cementing materials over the optimum amount may reduce the performance, both in strength and durability parameters. One of the purposes of a mixture experiment is to find the best proportion of each component and the best value of each process variable, in order to optimize a single response or multiple responses simultaneously. A comprehensive methodology for mixture experiment was first proposed by [3, 4]. Scheffe introduced the {q,m}simplex lattice design and simplex centroid designs. If the number of components is not large and a high order polynomial is needed in order to accurately describe the response surface; then, a simplex lattice design can be used [5]. Scheffe's model is most times referred to as mixture model. They differ from the usual regression model due to correlation among all components in the mixture designs. Another difference is that the intercept term in the model is not usually included in the regression model [6]. Scheffe expressed the functional relationship between the investigated property and mixture components. Scheffe's ideas endure as primary recourse for practitioners of mixture experiments [7]. In a bid to reduce air and water pollutions, global warming, cost of construction and environmental nuisance, some researchers have used oyster shell powder as supplementary cementitious material to produce ecologically and economically friendly concrete [8-13]. The use of supplementary cementing materials in concrete may help in reducing the large carbon dioxide emission that result from production of Portland cement [14]. However, none of those researchers was able to come up with a model to optimize these mixture components as a predictor of compressive strength, given the mix proportion and vice versa. Hence, the present study will focus on model formulation and validation of oyster shell powder-cement concrete using the Scheffe's simplex lattice theory.

2. Materials and methods

2.1 Materials

Dangote brand of ordinary Portland cement was used in this research and it conformed to the requirements of [15]. The sand was sourced from Imo River in Imo State. It was sieved through 10 mm British standards test sieve to remove cobbles. The sand was sharp and free from deleterious substances and conforms to the requirements of [16]. The granite was sourced from the quarry site at Ishiagu, Ebonyi State, Nigeria. The maximum size of aggregate used for this work is 20 mm diameter. It was thoroughly flushed with water to reduce the level of impurities and organic matter that might have intruded during quarrying; to conform to requirements [17]. The water used for the study was obtained from borehole. The water was clean and free from any visible impurities. It conformed to the requirements of [18]. The water does not contain harmful constituents in such quantities as may be detrimental to the setting, hardening and durability of the concrete. Oyster Shell Powder was obtained from oyster shells littered at Okwagwe River, Delta State, after washing, sun drying, crushing and sieving with 150 µm sieve. A total of 90 cubes of concrete were cast and cured for 28 days. 15 runs with three replicates each for the model compressive strength and 15 runs with three replicates each for validation of the model (control).

2.2 Method

2.2.1 Design of experiment

The OSP-cement concrete is made up of five components: water, cement, sand, granite and OSP which we can designate as X_1, X_2, X_3, X_4 an d X_5 respectively. Where, X_i represents the volume fraction of component. The volume fractions of the components sum to one, and the region defined by this constraint is the regular tetrahedron (or simplex) shown in *Fig. 1.*

Each vertex of the tetrahedron represents the pure component. For example, the vertex labelled X_1 is the pure water mixture with $X_1 = 1$, $X_2 = 0$, $X_3 = 0$, $X_4 = 0$ and $X_5 = 0$ or (1,0,0,0,0).

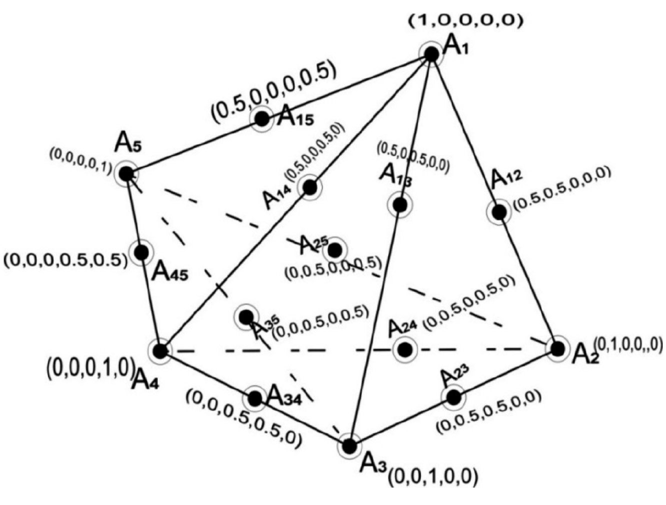

Fig. 1. A (5, 2) Scheffe's simplex lattice with 15 experimental runs
1. ábra (5,2) méretű Scheffe szimplex rács 15 vizsgálati ponttal

All responses (properties) of interest would be measured for each mixture in the design and modelled as a function of the components. Here, polynomial functions will be used. The number of coefficients, *n*, of the polynomial is determined using equation

$$n = \frac{(q+m-1)!}{(q-1)!*m!} \quad (1)$$

Where, *q* is the number of components of the mixture, and m is the degree of the polynomial. Thus, for *q=5* and *m=2* as in second degree polynomial, *n=15* signifying that for a (*q*,*m*) = (5,2) simplex design, we have 15 coefficients of the polynomial function; thus, 15 experimental runs.

The mixture constraint, according to [3] implies that

$$0 \le X_i \le 1, \text{ for } i = 1,2,\ldots,q \quad (2)$$

For a five – component mixture,

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \quad (3)$$

The linear polynomial model for a response y is

$$Y = b^*_o + b^*_1X_1 + b^*_2X_2 + b^*_3X_3 + b^*_4X_4 + b^*_5X_5 + e \quad (4)$$

Where b^*_i are constants and *e*, the random error term, represents the combined effects of all variables not included in the model. The form of Eq. (4) is called the Scheffe's linear mixture polynomial. If there is curvature in the system, a polynomial of higher degree such as the second order model should be sought. Hence, the quadratic model is given by Eq. (5).

$$Y = b^*_o + b^*_1X_1 + b^*_2X_2 + b^*_3X_3 + b^*_4X_4 + b^*_5X_5 + b^*_{12}X_1X_2 + b^*_{13}X_1X_3 + b^*_{14}X_1X_4 + b^*_{15}X_1X_5 + b^*_{23}X_2X_3 + b^*_{24}X_2X_4 + b^*_{25}X_2X_5 + b^*_{34}X_3X_4 + b^*_{35}X_3X_5 + b^*_{45}X_4X_5 + b^*_{11}X_1^2 + b^*_{22}X_2^2 + b^*_{33}X_3^2 + b^*_{44}X_4^2 + b^*_{55}X_5^2 + e \quad (5)$$

The challenge which any model developed using polynomial in Eq. (4) is that the developed model will always give an expected response, even when all the components are absent (zero). This limitation is due to the presence of b_o and e, random error in the polynomials [19]. Scheffe's model of Eq. (5) overcomes this weakness. Transformation of Eq. (5) gives Eq. (6) below

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5 \quad (6)$$

Eq. (6) is the regression equation for the (5, 2) Scheffe's second degree canonical polynomial. The determination of the values of the coefficients in Eq. (6) will complete the model equation. Where:

$$Y_1 = \beta_1,\ Y_2 = \beta_2,\ Y_3 = \beta_3,\ Y_4 = \beta_4,\ Y_5 = \beta_5,$$
$$\beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2,\ \beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3,$$
$$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4,\ \beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5,$$
$$\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3,\ \beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4,$$
$$\beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5,\ \beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4,$$
$$\beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5,\ \beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5 \quad (7)$$

2.2.2 Concrete mix ratios for the formulation of regression model

According to [20], the relationship between the actual and the pseudo mix ratios are given by

$$\{Z\} = [A]\{X\} \quad (8)$$

$$\{X\} = [A^{-1}]\{Z\} \quad (9)$$

Where *Z*, *A*, and *X* are respectively the real mix ratios, coefficient of relation matrix and pseudo mix ratios. The value of matrix A will be obtained from the first five real mix ratios. The first five mix ratios are: Z_1[0.45:0.95:2.00:4.00:0.05], Z_2[0.48:0.90:1.85:3.75:0.10], Z_3[0.5:0.85:2.15:4.15:0.15], Z_4[0.55:0.80:2.25:3.95:0.20], Z_5[0.60:0.75:1.75:3.65:0.25] and the corresponding pseudo mix ratios at the vertices of the tetrahedron (simplex) are X_1[1:0:0:0:0], X_2[0:1:0:0:0], X_3[0:0:1:0:0], X_4[0:0:0:1:0], X_5[0:0:0:0:0:1] as shown in *Fig. 1.*

This quadratic model involves 15 parameters, so design will be at least 15 points in order to fit the model. However, multiple points or complete replicates are required to provide sufficient degrees of freedom to test the adequacy of the fit.

Matrix *A* is the transpose of the first five mix ratios and these are as shown below:

$$[A]=\begin{vmatrix}0.45 & 0.48 & 0.5 & 0.55 & 0.60\\0.95 & 0.90 & 0.85 & 0.80 & 0.75\\2.00 & 1.85 & 2.15 & 2.25 & 1.75\\4.00 & 3.75 & 4.15 & 3.95 & 3.65\\0.05 & 0.10 & 0.15 & 0.20 & 0.25\end{vmatrix}$$

With the substitution of the other pseudo mix ratios at the midpoint of the simplex into Eq. (8), we obtain the 10 remaining real mix ratios. Hence, the mix ratios, both real and pseudo at the vertices and midpoints of the tetrahedron are as given in *Table 1*.

In order to statistically test the validity of the regression model for the compressive strength of OSP-cement concrete, fifteen additional mixes (control) were made as given in *Table 2*.

The results obtained from the 28-compressive strength for model formulation were fitted into the regression equation to form the regression model. The model was validated using the results of 28-day compressive strength obtained using the control mix ratios.

Points	Real mix ratios					Pseudo mix ratios				
	Water Z_1	Cement Z_2	Sand Z_3	Granite Z_4	OSP Z_5	Water X_1	Cement X_2	Sand X_3	Granite X_4	OSP X_5
Y_1	0.45	0.95	2.00	4.00	0.05	1.0	0.0	0.0	0.0	0.0
Y_2	0.48	0.90	1.85	3.75	0.10	0.0	1.0	0.0	0.0	0.0
Y_3	0.50	0.85	2.15	4.15	0.15	0.0	0.0	1.0	0.0	0.0
Y_4	0.55	0.8	2.25	3.95	0.20	0.0	0.0	0.0	1.0	0.0
Y_5	0.60	0.75	1.75	3.65	0.25	0.0	0.0	0.0	0.0	1.0
Y_{12}	0.465	0.925	1.925	3.875	0.075	0.5	0.5	0.0	0.0	0.0
Y_{13}	0.475	0.90	2.075	4.075	0.100	0.5	0.0	0.5	0.0	0.0
Y_{14}	0.500	0.875	2.125	3.975	0.125	0.5	0.0	0.0	0.5	0.0
Y_{15}	0.525	0.850	1.875	3.825	0.150	0.5	0.0	0.0	0.0	0.5
Y_{23}	0.490	0.875	2.000	3.950	0.125	0.0	0.5	0.5	0.0	0.0
Y_{24}	0.515	0.850	2.050	3.850	0.150	0.0	0.5	0.0	0.5	0.0
Y_{25}	0.540	0.825	1.800	3.700	0.175	0.0	0.5	0.0	0.0	0.5
Y_{34}	0.525	0.825	2.200	4.050	0.175	0.0	0.0	0.5	0.5	0.0
Y_{35}	0.550	0.800	1.950	3.900	0.200	0.0	0.0	0.5	0.0	0.5
Y_{45}	0.575	0.775	2.000	3.800	0.225	0.0	0.0	0.0	0.5	0.5

Table 1 Concrete mix ratios for model formulation
1. táblázat A modellalkotáshoz használt beton összetétel arányok

Point	Real mix ratios					Pseudo mix ratios				
	Water Z_1	Cement Z_2	Sand Z_3	Granite Z_4	OSP Z_5	Water X_1	Cement X_2	Sand X_3	Granite X_4	OSP X_5
C_1	0.495	0.875	2.0625	3.9625	0.1250	0.25	0.25	0.25	0.25	0.0
C_2	0.5075	0.8625	1.9375	3.8875	0.1375	0.25	0.25	0.25	0.0	0.25
C_3	0.5200	0.8500	1.9625	3.8375	0.1500	0.25	0.25	0.0	0.25	0.25
C_4	0.525	0.8375	2.0375	3.9375	0.1625	0.25	0.0	0.25	0.25	0.25
C_5	0.5325	0.8250	2.0000	3.8750	0.1750	0.0	0.25	0.25	0.25	0.25
C_6	0.5160	0.8500	2.000	3.9000	0.1500	0.2	0.2	0.2	0.2	0.2
C_7	0.4840	0.8900	2.0250	3.9650	0.1100	0.3	0.3	0.3	0.1	0.0
C_8	0.4890	0.8850	1.9750	3.9350	0.1150	0.3	0.3	0.3	0.0	0.1
C_9	0.5040	0.8700	2.0050	3.8750	0.1300	0.3	0.3	0.0	0.3	0.1
C_{10}	0.5100	0.8550	2.0950	3.9950	0.1450	0.3	0.0	0.3	0.3	0.1
C_{11}	0.5190	0.8400	2.0500	3.9200	0.1600	0.0	0.3	0.3	0.3	0.1
C_{12}	0.5400	0.8150	2.0450	3.9250	0.1850	0.1	0.0	0.3	0.3	0.3
C_{13}	0.534	0.8300	1.9550	3.8050	0.1700	0.1	0.3	0.0	0.3	0.3
C_{14}	0.519	0.8450	1.9250	3.8650	0.1550	0.1	0.3	0.3	0.0	0.3
C_{15}	0.504	0.8600	2.0750	3.9550	0.1400	0.1	0.3	0.3	0.3	0.0

Table 2 Concrete mix ratios (control) for model validation
2. táblázat A modell validálásához használt beton összetétel arányok

Response	Replicate	Average mass (kg)	Volume (m^3)	Crushing load (N)	Compressive strength (N/mm^2)	Average compressive strength (N/mm^2)
Y1	A	8.20	0.003375	400000	17.78	19.48
Y1	B			490000	21.78	
Y1	C			425000	18.89	
Y2	A	8.72	0.003375	590000	26.22	29.01
Y2	B			730000	32.44	
Y2	C			638000	28.36	
Y3	A	8.28	0.003375	450000	20.00	19.56
Y3	B			430000	19.11	
Y3	C			440000	19.56	
Y4	A	8.38	0.003375	500000	22.22	21.93
Y4	B			460000	20.44	
Y4	C			520000	23.11	
Y5	A	8.33	0.003375	400000	17.78	19.21
Y5	B			482000	21.42	
Y5	C			415000	18.44	
Y12	A	8.48	0.003375	650000	28.89	30.81
Y12	B			730000	32.44	
Y12	C			700000	31.11	
Y13	A	8.68	0.003375	700000	31.11	27.41
Y13	B			550000	24.44	
Y13	C			600000	26.67	
Y14	A	8.77	0.003375	670000	29.78	25.93
Y14	B			530000	23.56	
Y14	C			550000	24.44	
Y15	A	8.70	0.003375	580000	25.78	24.74
Y15	B			570000	25.33	
Y15	C			520000	23.11	
Y23	A	8.77	0.003375	505000	22.44	23.48
Y23	B			535000	23.78	
Y23	C			545000	24.22	
Y24	A	8.77	0.003375	615000	27.33	26.59
Y24	B			585000	26.00	
Y24	C			595000	26.44	
Y25	A	8.63	0.003375	540000	24.00	23.70
Y25	B			510000	22.67	
Y25	C			550000	24.44	
Y34	A	8.18	0.003375	455000	20.22	17.85
Y34	B			385000	17.11	
Y34	C			365000	16.22	
Y35	A	8.15	0.003375	440000	19.56	18.89
Y35	B			420000	18.67	
Y35	C			415000	18.44	
Y45	A	8.42	0.003375	425000	18.89	18.30
Y45	B			402000	17.87	
Y45	C			408000	18.13	

Table 3 The 28-day compressive strength values for model formulation
3. táblázat A model alkotáshoz használt 28 napos nyomószilárdság értékek

Response (control)	Replicate	Average weight (kg)	Volume (m^3)	Crushing load (N)	Compressive strength (N/mm^2)	Average compressive strength (N/mm^2)
C1	A	8.52	0.003375	480000	21.33	21.85
C1	B			500000	22.22	
C1	C			495000	22.00	
C2	A	7.45	0.003375	640000	28.44	24.59
C2	B			510000	22.67	
C2	C			510000	22.67	
C3	A	8.45	0.003375	540000	24.00	22.59
C3	B			470000	20.89	
C3	C			515000	22.89	
C4	A	8.68	0.003375	610000	27.11	24.59
C4	B			525000	23.33	
C4	C			525000	23.33	
C5	A	8.65	0.003375	550000	24.44	24.44
C5	B			580000	25.78	
C5	C			520000	23.11	
C12	A	8.33	0.003375	540000	24.00	25.26
C12	B			580000	25.78	
C12	C			585000	26.00	
C13	A	8.20	0.003375	575000	25.56	25.93
C13	B			580000	25.78	
C13	C			595000	26.44	
C14	A	8.57	0.003375	570000	25.33	26.67
C14	B			620000	27.56	
C14	C			610000	27.11	
C15	A	8.47	0.003375	560000	24.89	24.79
C15	B			595000	26.44	
C15	C			518000	23.02	
C23	A	8.07	0.003375	505000	22.44	22.67
C23	B			510000	22.67	
C23	C			515000	22.89	
C24	A	8.07	0.003375	457000	20.31	19.14
C24	B			420000	18.67	
C24	C			415000	18.44	
C25	A	8.23	0.003375	340000	15.11	19.04
C25	B			495000	22.00	
C25	C			450000	20.00	
C34	A	8.33	0.003375	470000	20.89	19.73
C34	B			462000	20.53	
C34	C			400000	17.78	
C35	A	8.47	0.003375	490000	21.78	21.11
C35	B			505000	22.44	
C35	C			430000	19.11	
C45	A	8.23	0.003375	410000	18.22	18.30
C45	B			380000	16.89	
C45	C			445000	19.78	

Table 4 The 28-day compressive strength values for model validation (control)
4. táblázat A model validálásához használt 28 napos nyomószilárdság értékek

β_1	β_2	β_3	β_4	β_5	β_{12}	β_{13}	β_{14}	β_{15}	β_{23}	β_{24}	β_{25}	β_{34}	β_{35}	β_{45}
19.48	29.01	19.56	21.93	19.21	26.28	31.56	20.89	21.57	-3.20	4.50	-1.63	-11.56	-1.99	-9.10

Table 5. Coefficients of Scheffe's second degree polynomial for the regression model
5. táblázat Scheffe másodfokú polinomjának együtthatói a regressziós modellhez

3. Regression model for the compressive strength of OSP-cement concrete

The results of 28-day compressive strength for model formulation of OSP-Cement concrete are a given in *Table 3*.

Similarly, the results of 28-day compressive strength for model validation of OSP-cement concrete are a given in *Table 4*.

From Eq. (7), the coefficients of the Scheffe's second degree polynomial are given in *Table 5*.

Substituting the values of these coefficients into Eq. (6) yields $\hat{Y} = 19.48X_1 + 29.01X_2 + 19.56X_3 + 21.93X_4 + 19.21X_5 + 26.28X_1X_2 + 31.56X_1X_3 + 20.89X_1X_4 + 21.57X_1X_5 - 3.20X_2X_3 + 4.50X_2X_4 - 1.63X_2X_5 - 11.56X_3X_4 - 1.99X_3X_5 - 9.10X_4X_5$ (10)

Eq. (10) is the regression model for the compressive strength of oyster shell powder-cement concrete using the Scheffe's simplex lattice theory.

4. Model Validations

4.1 Replication Variance

Mean responses, Y and the variances of replicates, S_i^2 in *Table 9* were obtained from Eq. (11).

$$Y = \frac{\sum_{i=1}^{n} Y_i}{n} \quad (11)$$

$$S_i^2 = \left[\frac{1}{n-1}\right]\left[\sum Y_i^2 \left[\frac{1}{n(\sum Y_i)^2}\right]\right] \quad (12)$$

Where: $1 \leq i \leq n$

Expansion of Eq. (12) gives Eq. (13)

$$S_i^2 = \left[\frac{1}{n-1}\right]\left[\sum_{i=n}^{n}[Y_i - Y]^2\right] \quad (13)$$

Where Y_i = responses; Y = mean of the responses for each control point; n = number of parallel observations at every point; $n - 1$ = degree of freedom; Si^2 = variance at each design point. For all the design points, N, the degree of freedom, V_e is given by

$$V_e = \sum N - 2 = 30 - 2 = 28 \quad (14)$$

Where:
N is the number of points.
$S_y^2 = 103.24/28 = 3.687$
Where S_i^2 is the variance at each point
$S_y = 1.92$

The results of the compressive strength obtained for the formulation and validation of the model based on Scheffe's lattice theory are given in *Table 7*.

4.2 Test of adequacy of the model

The test for adequacy of the model was done using Student's t-test at 95% confidence level on the compressive strength at the control points subject to these two hypotheses.

Null hypothesis

There is no significant difference between the laboratory tests and model predicted strength results.

Alternative hypothesis

There is a significant difference between the laboratory test and model predicted strength results.

4.2.1 Student's t-test

Table 8 shows the parameters with which the student's t-test will be done. We did a two-tailed test (inequality) and if $t_{Stat} > t_{Critical}$ two-tailed, we reject the null hypothesis.

$$t_{Stat} = \frac{\sum(\text{lab}-\text{model})}{\sqrt{\frac{(15*\sum((\text{lab}-\text{model})^2) - (\sum(\text{lab}-\text{model}))^2}{(15-1)}}} =$$

$$\frac{(16.61)}{\sqrt{\frac{(15*92.24)-(16.61)}{(15-1)}}} = 1.868$$

α = 0.05 and 0.025 for two tail; DF = 15-1 = 14 (t-distribution table).

$t_{Critical} = 2.145$

$t_{Stat} < t_{Critical}$

From the calculations, $t_{Stat} = 1.868$ and $t_{Critical}$ two-tailed = 2.145, so $t_{Critical} > t_{Stat}$. Therefore, we accept the null hypothesis.

5. Conclusions

The present study was geared towards the formulation and validation of model to predict the compressive strength of OSP-cement concrete, given the mix proportions and vice versa, based on Scheffe's simplex lattice theory. From the foregoing results, the following conclusions are hereby drawn:

i. The statistical tests conducted to validate the Scheffe's lattice model formulated for OSP-cement blended concrete for 28-day compressive strength was found to be adequate for the model.

ii. A good agreement was found between the predicted and experimental values of the 28-day compressive strength, leading to the adoption of null hypothesis.

iii. The model gave highest compressive strength of 30.81 N/mm² corresponding to mix ratio of 0.54:0.815:2.045:3.925:0.185 for water, cement, sand, granite and OSP respectively and the minimum compressive strength of 17.85 N/mm² corresponding to mix ratio of 0.525:0.825:2.2:4.05:0.175 for water, cement, sand, granite and OSP respectively.

iv. Using the model, compressive strength of all points in the simplex can be derived.

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Response	Replicate	Response Y_i (N/mm²)	$\sum Y_i$	Y	$\sum Y_i^2$	S_i^2
Y1	A	17.78	58.44	19.48	1147.11	4.26
Y1	B	21.78				
Y1	C	18.89				
Y2	A	26.22	87.02	29.01	2544.28	10.00
Y2	B	32.44				
Y2	C	28.36				
Y3	A	20.00	58.67	19.56	1147.65	0.20
Y3	B	19.11				
Y3	C	19.56				
Y4	A	22.22	65.78	21.93	1445.93	1.84
Y4	B	20.44				
Y4	C	23.11				
Y5	A	17.78	57.64	19.21	1115.16	3.77
Y5	B	21.42				
Y5	C	18.44				
Y12	A	28.89	92.44	30.81	2855.11	3.23
Y12	B	32.44				
Y12	C	31.11				
Y13	A	31.11	82.22	27.41	2276.54	11.52
Y13	B	24.44				
Y13	C	26.67				
Y14	A	29.78	77.78	25.93	2039.11	11.33
Y14	B	23.56				
Y14	C	24.44				
Y15	A	25.78	74.22	24.74	1840.40	2.04
Y15	B	25.33				
Y15	C	23.11				
Y23	A	22.44	70.44	23.48	1655.85	0.86
Y23	B	23.78				
Y23	C	24.22				
Y24	A	27.33	79.78	26.59	2122.42	0.46
Y24	B	26.00				
Y24	C	26.44				
Y25	A	24.00	71.11	23.70	1687.31	0.86
Y25	B	22.67				
Y25	C	24.44				
Y34	A	20.22	53.56	17.85	964.89	4.41
Y34	B	17.11				
Y34	C	16.22				
Y35	A	19.56	56.67	18.89	1071.06	0.35
Y35	B	18.67				
Y35	C	18.44				
Y45	A	18.89	54.89	18.30	1004.83	0.28
Y45	B	17.87				
Y45	C	18.13				

C1	A	21.33	65.56	21.85	1432.94	0.21
C1	B	22.22				
C1	C	22.00				
C2	A	28.44	73.78	24.59	1836.64	11.13
C2	B	22.67				
C2	C	22.67				
C3	A	24.00	67.78	22.59	1536.25	2.49
C3	B	20.89				
C3	C	22.89				
C4	A	27.11	73.78	24.59	1823.90	4.76
C4	B	23.33				
C4	C	23.33				
C5	A	24.44	73.33	24.44	1796.15	1.78
C5	B	25.78				
C5	C	23.11				
C12	A	24.00	75.78	25.26	1916.49	1.20
C12	B	25.78				
C12	C	26.00				
C13	A	25.56	77.78	25.93	2016.89	0.21
C13	B	25.78				
C13	C	26.44				
C14	A	25.33	80.00	26.67	2136.10	1.38
C14	B	27.56				
C14	C	27.11				
C15	A	24.89	74.36	24.79	1848.79	2.94
C15	B	26.44				
C15	C	23.02				
C23	A	22.44	68.00	22.67	1541.43	0.05
C23	B	22.67				
C23	C	22.89				
C24	A	20.31	57.42	19.14	1101.18	1.04
C24	B	18.67				
C24	C	18.44				
C25	A	15.11	57.11	19.04	1112.35	12.56
C25	B	22.00				
C25	C	20.00				
C34	A	20.89	59.20	19.73	1174.01	2.90
C34	B	20.53				
C34	C	17.78				
C35	A	21.78	63.33	21.11	1343.26	3.11
C35	B	22.44				
C35	C	19.11				
C45	A	18.22	54.89	18.30	1008.44	2.09
C45	B	16.89				
C45	C	19.78				
						Σ = 103.24

Table 6 Experimental test results and the replication variance

6. táblázat Kísérleti eredmények és az ismétlési variancia

Symbol	Experimental test	Scheffe's model
Y1	19.48	19.48
Y2	29.01	29.01
Y3	19.56	19.56
Y4	21.93	21.93
Y5	19.21	19.21
Y12	30.81	30.81
Y13	27.41	27.41
Y14	25.93	25.93
Y15	24.74	24.74
Y23	23.48	23.48
Y24	26.59	26.59
Y25	23.70	23.70
Y34	17.85	17.85
Y35	18.89	18.89
Y45	18.30	18.30
C1	21.85	23.14
C2	24.59	23.01
C3	22.59	21.36
C4	24.59	22.44
C5	24.44	17.97
C12	25.26	21.45
C13	25.93	24.20
C14	26.67	24.14
C15	24.79	22.56
C23	22.67	23.50
C24	19.14	18.60
C25	19.04	20.23
C34	19.73	18.95
C35	21.11	21.39
C45	18.30	21.14

Table 7 Experimental test and Scheffe's model results
7. táblázat Kísérleti és Scheffe modell eredmények

Symbol	Lab	Model	Lab-Model	(Lab-Model) ^ 2
C1	21.85	23.14	-1.29	1.67
C2	24.59	23.01	1.58	2.50
C3	22.59	21.36	1.23	1.52
C4	24.59	22.44	2.15	4.61
C5	24.44	17.97	6.48	41.94
C12	25.26	21.45	3.81	14.51
C13	25.93	24.20	1.73	2.98
C14	26.67	24.14	2.53	6.39
C15	24.79	22.56	2.23	4.96
C23	22.67	23.50	-0.83	0.70
C24	19.14	18.60	0.54	0.29
C25	19.04	20.23	-1.20	1.43
C34	19.73	18.95	0.78	0.61
C35	21.11	21.39	-0.27	0.08
C45	18.30	21.14	-2.84	8.06
	Total		**16.61**	**92.24**

Table 8 Student's t-test for the compressive strength
8. táblázat A nyomószilárdság Student tényező (t) változásának tesztje

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