VIRTUAL MODELS OF MACHINE TOOLS WITH PARALLEL TOPOLOGY

Párhuzamos szerkezetű szerszámgépek virtuális modelljei

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Tartalom

A dolgozat a párhuzamos robotok modelljének alapelemeit mutatja be, melyeket a modern szerszámgépek szerkezeti felépítésénél használnak fel. Ezen új típusú szerszámgépek előnye a hagyományosokkal szemben a nagyobb pontosság, magasabb hatásfok, stb. biztosítása, ami sajátos, zárt kinematikai láncának köszönhető. Az alábbiakban a párhuzamos szerkezetű szerszámgépek működését mutatjuk be virtuális modelleken.

A modellek különböző jellemzők szerint csoportosíthatók, mint például a tehetetlenségi erők, az összetevő alkatrészek rugalmassága, stb. Mivel a kinematikai és dinamikai paraméterek numerikusan meghatározhatók, ezen modellek a mérő- és vezérlőegységek tervezésének alapelemeit képezik.

Keywords: Parallel Robot, Kinematics, Modular Design, Reconfigurable Topology, Virtual modeling

1. Introduction

A good dynamic behavior (high stiffness), a high accuracy and a good ratio between total mass and manipulated mass are just few advantages of parallel robots compared with serial type. However, the design, trajectory planning and application development of the parallel robot are difficult and tedious because the closed-loop mechanism leads to complex kinematics. To overcome this drawback, modular design concept is introduced in the development of parallel robots. Also, during the last period new types of applications were developed. These new applications are related to the machine tools with parallel topology. Utilization of parallel topology in the machine tools field creates the possibility for a reconfigurable design which is still an open problem and lacks theoretical base. One of the problems for reconfigurable robots is to determine the topology and geometry of the robot which is the suitable to fulfil a set of criteria. In the following sections we first present the modular topologic synthesis. Then, we describe the kinematics and an example is given.

2. Modular topologic synthesis

The structural synthesis of parallel mechanisms could be made if the relation of the number of degrees of freedom it is considered :

$$M = (6-m)\cdot n - \sum_{k=1}^{5}(k-m)\cdot C_k - M_P \tag{1}$$

where m is the number of common restrictions for all elements, n is the number of the mobile elements, k is the number of restrictions which define a joint (for example in the case of prismatic joint k=5), C_k is the number of joints with (6-k) degrees of freedom and M_P is the number of identical degrees of freedom.

In the case of parallel mechanisms without common restrictions and also without identical degrees of freedom the relation (1) it becomes:

$$M = 6\cdot n - \sum_{k=1}^{5} k\cdot C_k \ . \tag{2}$$

Let be N the number of mobile platforms and D_k – the number of joints with (6-k) degrees of freedom which directly connect the platforms of the mechanism.

With these notations it results:

$$M = 6 \cdot (n_1 + N) - \sum_{k=1}^{5} k \cdot (C_k + D_k) \quad (3)$$

where n_1 is the number of the elements which compose the loops which connect the platforms of the mechanism.

We can also assume (Fig.1) eight types of basic modules (named basic legs) which can connect the platforms of the mechanism.

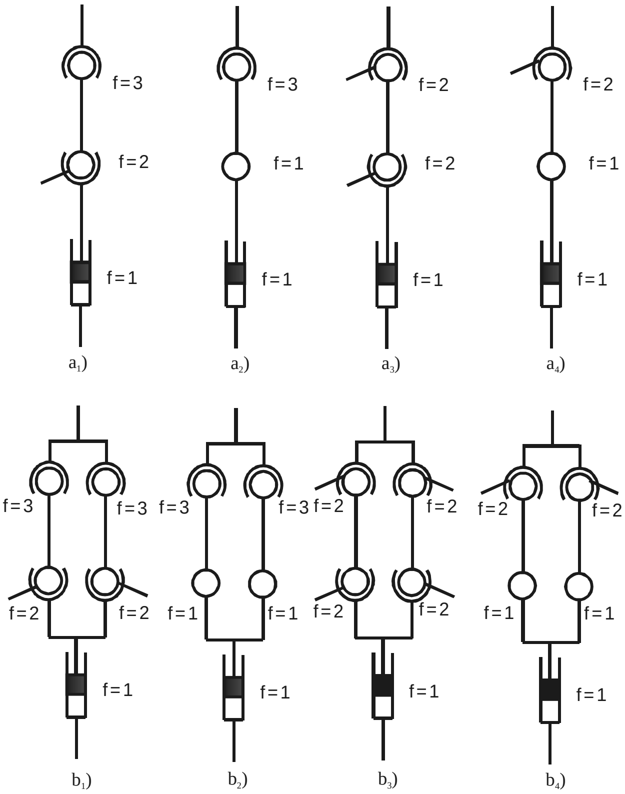

Fig. 1

Let be:

1] a_1 the number of the loops with prismatic – universal – spherical (PUS) topology,
2] a_2 the number of the loops with prismatic – rotational – spherical (PRS) topology,
3] a_3 the number of the loops with prismatic – universal – universal (PUU) topology,
4] a_4 the number of the loops with prismatic – rotational – universal (PRU) topology,
5] b_1 the number of the loops with prismatic – two universal – two spherical (P2U2S) topology,
6] b_2 the number of the loops with prismatic – two rotational – two spherical (P2R2S) topology,
7] b_3 the number of the loops with prismatic – two universal – two universal (P2U2U) topology,
8] b_4 the number of the loops with prismatic – two rotational –two universal (P2R2U) topology.

With these notations, the relation (3) it becomes:

$$M = 6 \cdot N - \sum_{k=1}^{5} k \cdot D_k - a_2 - a_3 - 2 \cdot a_4 - b_1 - 3 \cdot b_2 - 3 \cdot b_3 - 5b_4 \quad (4)$$

In the case of parallel mechanisms which are used in the field of machine tools, it is common to consider:

$$N = 1,\ D_k = 0,\ k = \{1,...,5\},\ a_3 = a_4 = b_3 = b_4 = 0 \quad (5)$$

Replacing (5) in (4) it results:

$$M = 6 - a_2 - b_1 - 3 \cdot b_2 \tag{6}$$

Because each loop contain only one degrees of freedom, we can write:

$$M = a_1 + a_2 + b_1 + b_2 \tag{7}$$

From (7) it results:

$$a_1 = M - a_2 - b_1 - b_2 \tag{8}$$

Integer solutions of the equations:

$$\begin{aligned} &M - 6 + a_2 + b_1 + 3 \cdot b_2 = 0, \\ &a_1 - M + a_2 + b_1 + b_2 = 0 \end{aligned} \tag{9}$$

gives all variants of parallel mechanisms with assumed hypothesis. For example if M = 6, it results:

$$a_1 = 6\,, a_2 = b_1 = b_2 = 0. \tag{10}$$

The relations (10) define the Stewart Platform. The system of equations (9) has many solutions. Also, if other parameters are taken into consideration (the order of the joints in the loop, the geometrical parameters of the loops etc) the topology problem becomes very complex.

The relation (4) define the topology of parallel robots in a modular manner. If other parameters are taken into consideration (the order of the joints in the loop, the geometrical parameters of the loops etc) the topology problem becomes very complex.

3. Kinematics

General algorithms used to solve direct kinematics in the case of parallel mechanisms consider that for each independent loop of the mechanism one vector equation can be write. Thus, a nonlinear system of scalar equations is obtained. Usually, this system of equations can be solved only with numerical methods and for that an accurate initial value of the solution it is required. Of course, this initial value of the solution is strongly related to the geometric parameters of the mechanism. When the geometric parameters of the mechanism are changed also the initial solution must be changed. According to that, the kinematics of the parallel mechanism will be developed in a modular manner, based on kinematics of the legs which connect the platforms and in order to ensure an analytical value for the initial solution. Each leg is in fact the right (or left) side of one independent closed loop and can be described by two coordinate systems: one attached to the frame and the other one attached to the mobile platform (Fig. 2).

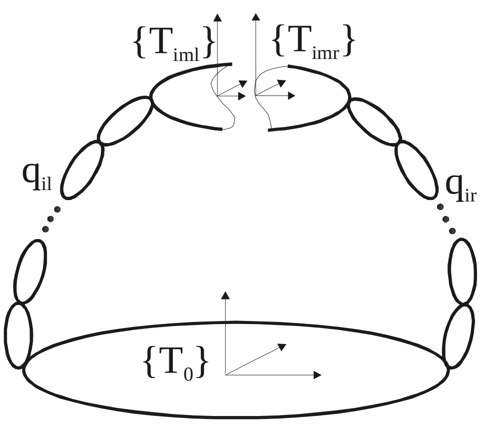

Fig 2

The relationship between these coordinate systems is given by:

$$\mathbf{H}_{\mathrm{iml}} = \prod \mathbf{A}_{\mathrm{il}}(\mathrm{q}_{\mathrm{il}})\,, \tag{11}$$

for the left part of the independent loop and :

$$\mathbf{H}_{\mathrm{imr}} = \prod \mathbf{A}_{\mathrm{ir}}(\mathrm{q}_{\mathrm{ir}})\,, \tag{12}$$

for the right part.

$\mathbf{H}_{\mathrm{iml}}$, $\mathbf{H}_{\mathrm{imr}}$ are absolute transformation matrices and $\mathbf{A}_{\mathrm{il}}(\mathrm{q}_{\mathrm{il}})$, $\mathbf{A}_{\mathrm{ir}}(\mathrm{q}_{\mathrm{ir}})$ are relative transformation matrices.

For an independent loop it results:

$$\mathbf{H}_{\mathrm{iml}} = \mathbf{H}_{\mathrm{imr}} \tag{13}$$

Matrix equation (13) leads to six independent scalar equations. For whole parallel mechanism, a nonlinear system of equations (with 6n independent scalar equations, where n is the number of independent loops) will be obtained. This system of equations can be solved only with numerical methods. Generally, the legs of the parallel component have the same topology. It results that the relative transformation matrices for the left and right part of each loop are formal similar. Therefore, for each topology of the legs, a formal mathematical entity (named **LMM - L**eg **M**athematical **M**odel) can be developed. Similarly a modular kinetostatic model can be developed. This mathematical model leads to non-linear system of equations. Classic algorithms of numerical methods, e.g. Newton-Raphson, can be used in order to solve this system of equations.

Usually a virtual model must be designed in order to ensure a friendly way to cooperate with the customer. Related to the virtual parallel mechanisms and in order to ensure this property, the virtual model of LMM must include an automatic way to find an initial solution for the nonlinear system of equations.

Without losing the generality of the problem, a leg with PSU topology is considered (Fig.3). An analytical solution of the initial values of the angular parameters of the joints of the leg means that a solution of the inverse geometric model for the initial position must be determined. This solution is also the initial solution for the nonlinear system of equations for the whole mechanism.

Thus, for the leg from figure 3 it results:

$$\begin{aligned}
&\mathbf{H} = \mathbf{A_1} \cdots \mathbf{A_6} \\
&\mathbf{A^{-1}}_1\ \mathbf{H} = \mathbf{A_2} \cdots \mathbf{A_6} \\
&\mathbf{A^{-1}}_2\ \mathbf{A^{-1}}_1\ \mathbf{H} = \mathbf{A_3} \cdots \mathbf{A_6} \\
&\mathbf{A^{-1}}_3\ \mathbf{A^{-1}}_2\ \mathbf{A^{-1}}_1\ \mathbf{H} = \mathbf{A_4} \cdots \mathbf{A_6} \\
&\mathbf{A^{-1}}_4\ \mathbf{A^{-1}}_3\ \mathbf{A^{-1}}_2\ \mathbf{A^{-1}}_1\ \mathbf{H} = \mathbf{A_5} \cdot \mathbf{A_6} \\
&\mathbf{A^{-1}}_5\ \mathbf{A^{-1}}_4\ \mathbf{A^{-1}}_3\ \mathbf{A^{-1}}_2\ \mathbf{A^{-1}}_1\ \mathbf{H} = \mathbf{A_6}
\end{aligned} \tag{14}$$

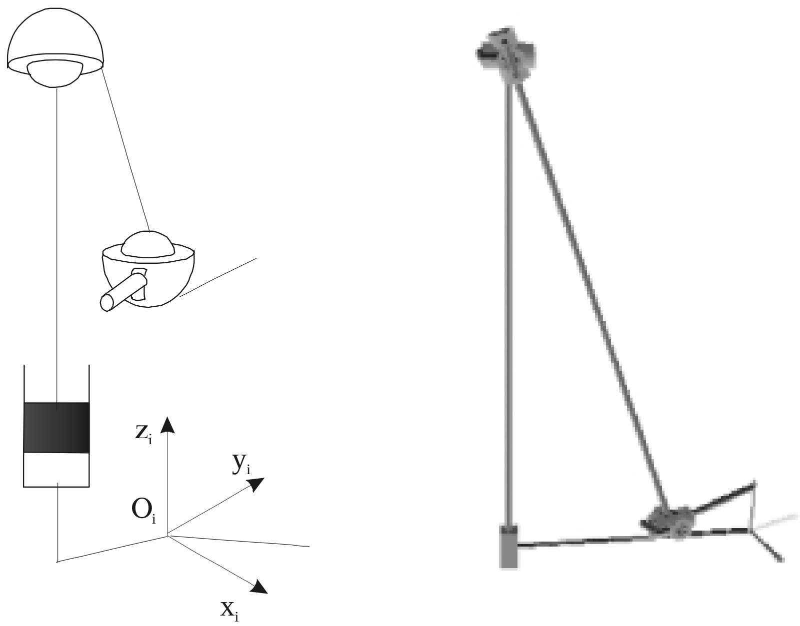

Fig 3

where H is the absolute transformation matrix, which describes the absolute position and orientation of the mobile platform (known for the initial position of the mechanism), A_i (i=1,6) is the relative transformation matrix. The elements of the A_i are functions of the joint coordinate (q_i for the prismatic joint and α_{ij} for all other joints of the leg). Using relations (14) a set of initial values for the parameters which describe the leg from figure 3 can be found.

Example

The mechanism shown in figure 4 as an example, has three degrees of freedom and five independent kinematic loops.

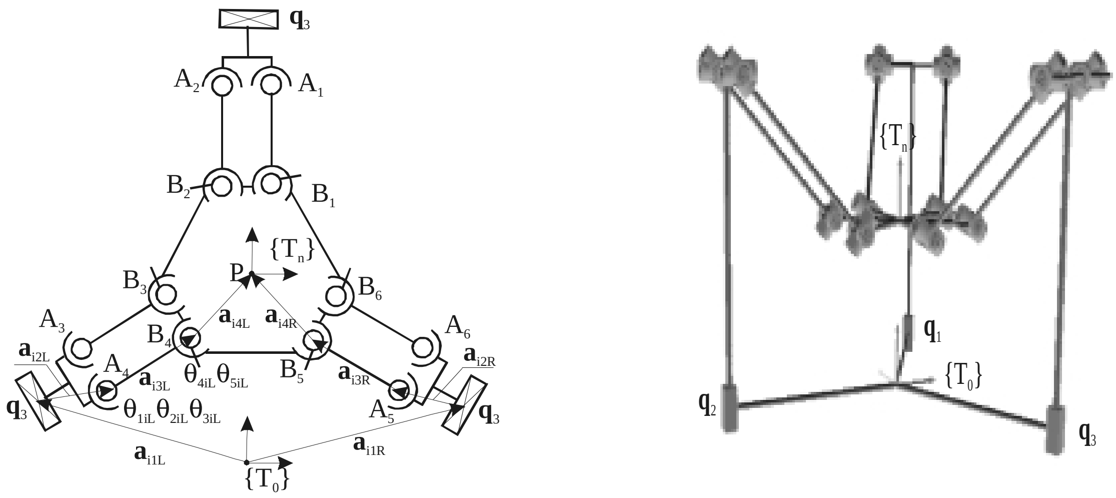

Fig. 4

Thus, for each closed independent loop, the closing equations are:

$$\begin{aligned} \sum_{j=1}^{4} \mathbf{a}_{ijL} &= \sum_{j=1}^{4} \mathbf{a}_{ijR} \ , \\ \mathbf{R}_{iL} &= \mathbf{R}_{iR} \end{aligned} \tag{17}$$

where $\mathbf{R}_{iL}$ and $\mathbf{R}_{iR}$ are the absolute orientation matrices, corresponding to the left and right side respectively of the closed independent loop.

The system of equations described by (17) contains (in the case of all five loops) 30 unknowns.

These are the angular displacements (θ_{jiL} and θ_{jiR}) at the level of universal and spherical joints respectively. This system of equations can be solved with numerical methods. In order to find an initial solution (necessary for numerical methods) classical algorithm of inverse kinematic applied for an open loop, which connect the mechanism platforms, can be used:

$$\prod_{j=1}^{k} \mathbf{A}^{-1}{}_{jiL}(\theta_{jiL0})\, \mathbf{H}_0 = \prod_{j=k+1}^{6} \mathbf{A}_{jiL}(\theta_{jiL0}). \tag{18}$$

where H_0 is the absolute transformation matrix and A_{jiL} are the relative transformation matrices.

Figure 4 (b) is the simulation in MOBILE software package.

4. Conclusions

The conclusion can be drawn as follows:

- Based on assumed modules and on relation of the number of degrees of freedom for a mechanism, a topologic synthesis can be done.
- The kinematics of the whole mechanism can be developed on a modular manner, each module based on the kinematics of one leg.
- Solving inverse kinematics of one leg it is possible to find an analytical solution of the initial value of the solution of the system of equations, which solve the direct kinematics of the mechanism.
- An analytical solution for the initial value of the solution of the system of equations corresponding to the direct kinematic of the mechanism increases significantly flexibility of the simulation model. Thus, it is possible to change automatically and interactive the geometric parameters of the mechanism during the simulation.

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