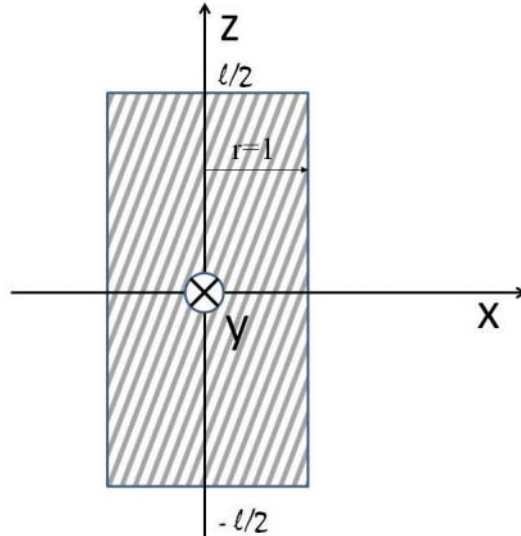
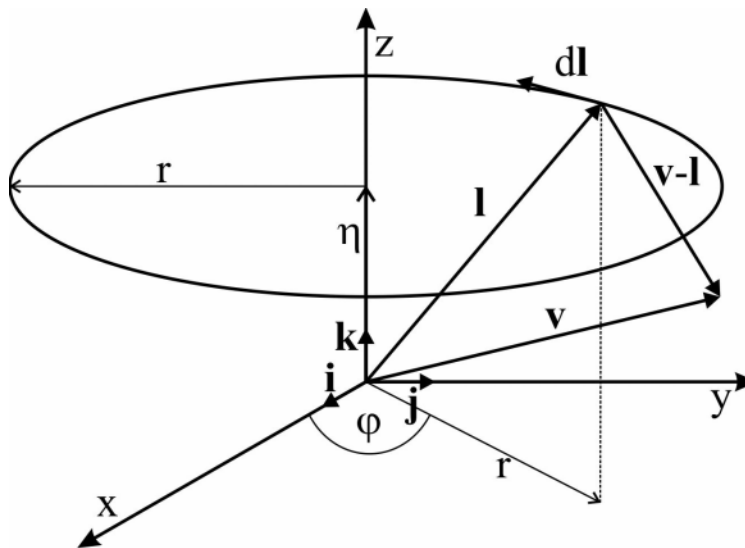


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A mágneses vektorpotenciál, mint valóságosan létező vektormező
 – melléklet –

A szolenoid mágneses vektorpotenciál (**A**) terének és mágneses indukció (**B**) terének számítása:



A szolenoid metszete. Forgástengelye a z tengely, míg x és y a forgástengelyre merőleges síkot képez.



A vektorpotenciál teret a körvezetők terének integrálásával kapjuk.

$$\mathbf{A}^*(\mathbf{v}) = \frac{\mu_0}{4\pi} \int \frac{I^* d\mathbf{l}}{|\mathbf{v} - \mathbf{l}|}$$

$$\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{l} = r \cos \varphi \mathbf{i} + r \sin \varphi \mathbf{j} + \eta \mathbf{k}$$

$$\frac{d\mathbf{l}}{d\varphi} = \mathbf{i}(-r \sin \varphi) + \mathbf{j}r \cos \varphi$$

$$d\mathbf{l} = [\mathbf{i}(-r \sin \varphi) + \mathbf{j}r \cos \varphi]d\varphi$$

$$\mathbf{v} - \mathbf{l} = (x - r \cos \varphi)\mathbf{i} + (y - r \sin \varphi)\mathbf{j} + (z - \eta)\mathbf{k}$$

$$|\mathbf{v} - \mathbf{l}|^2 = D = (x - r \cos \varphi)^2 + (y - r \sin \varphi)^2 + (z - \eta)^2 = x^2 + y^2 + (z - \eta)^2 + r^2 - 2r(x \cos \varphi + y \sin \varphi)$$

$$D = x^2 + y^2 + (z - \eta)^2 + r^2 - 2r(x \cos \varphi + y \sin \varphi)$$

$$\mathbf{A}^*(x, y, z) = \frac{\mu_0}{4\pi} \int \frac{I^* d\mathbf{l}}{\sqrt{D}}$$

$$\mathbf{A}^*(x, y, z) = \frac{\mu_0 I^*}{4\pi} \left(\mathbf{i} \int_{-\pi}^{\pi} \frac{-r \sin \varphi}{\sqrt{D}} d\varphi + \mathbf{j} \int_{-\pi}^{\pi} \frac{r \cos \varphi}{\sqrt{D}} d\varphi \right)$$

$$I^* = \frac{NI}{l} d\eta$$

$$d\mathbf{A}(x, y, z) = \frac{\mu_0 NI}{4\pi l} d\eta \left(\mathbf{i} \int_{-\pi}^{\pi} \frac{-r \sin \varphi}{\sqrt{D}} d\varphi + \mathbf{j} \int_{-\pi}^{\pi} \frac{r \cos \varphi}{\sqrt{D}} d\varphi \right)$$

$$\mathbf{A}(x, y, z) = \frac{\mu_0 NI}{4\pi l} r \left[\mathbf{i} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi}{\sqrt{D}} d\varphi \right) d\eta + \mathbf{j} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\eta \right]$$

$$A_{\max} = \mu_0 \frac{NI}{l} \frac{r}{2}$$

$$\frac{\mathbf{A}(x, y, z)}{A_{\max}} = \frac{1}{2\pi} \left[\mathbf{i} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi}{\sqrt{D}} d\varphi \right) d\eta + \mathbf{j} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\eta \right]$$

Komponensek szerint:

$$\frac{A_x(x, y, z)}{A_{\max}} = \frac{1}{2\pi} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi}{\sqrt{D}} d\varphi \right) d\eta$$

$$\frac{A_y(x, y, z)}{A_{\max}} = \frac{1}{2\pi} \int_{-l/2}^{l/2} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\eta$$

Forgásszimmetria miatt x, z sík metszetet vizsgálunk. Itt $y = 0$

$$D = x^2 + (z - \eta)^2 + r^2 - 2rx \cos \varphi \quad \text{páros fv}$$

$$\frac{A_x(x, z)}{A_{\max}} = \frac{1}{2\pi} \int_{-1/2}^{1/2} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi}{\sqrt{D}} d\varphi \right) d\eta \quad \text{Az integrandus páratlan fv} \quad A_x(x, z) = 0$$

$$\frac{A_y(x, z)}{A_{\max}} = \frac{1}{2\pi} \int_{-1/2}^{1/2} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\eta \quad \text{Az integrandus páros fv tehát} \quad A_y(x, z) \neq 0.$$

Általánosságban tehát:

$$\mathbf{A}(x, y, z) = \frac{A_{\max}}{2\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{-\sin \varphi}{\sqrt{D}} + \mathbf{j} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \right] d\eta$$

A mágneses indukció vektorát a mágneses vektorpotenciál rotációjaként kapjuk:

$$\mathbf{B} = \text{rot} \mathbf{A}$$

$$\mathbf{B}(x, y, z) = \frac{A_{\max}}{2\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \text{rot} \left(\mathbf{i} \frac{-\sin \varphi}{\sqrt{D}} + \mathbf{j} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \right] d\eta$$

Vizsgáljuk az integrandus vektort!

$$\text{rot} \left(\mathbf{i} \frac{-\sin \varphi}{\sqrt{D}} + \mathbf{j} \frac{\cos \varphi}{\sqrt{D}} \right) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{\sin \varphi}{\sqrt{D}} & \frac{\cos \varphi}{\sqrt{D}} & 0 \end{vmatrix} =$$

$$= -\mathbf{i} \frac{\partial}{\partial z} \left(\frac{\cos \varphi}{\sqrt{D}} \right) - \mathbf{j} \frac{\partial}{\partial z} \left(\frac{\sin \varphi}{\sqrt{D}} \right) + \mathbf{k} \left[\frac{\partial}{\partial x} \left(\frac{\cos \varphi}{\sqrt{D}} \right) + \frac{\partial}{\partial y} \left(\frac{\sin \varphi}{\sqrt{D}} \right) \right]$$

Az \mathbf{i} komponens:

$$-\frac{\partial}{\partial z} \left(\frac{\cos \varphi}{\sqrt{D}} \right) = -\cos \varphi \frac{\partial}{\partial z} \left(D^{-1/2} \right) = -\cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial z} = \cos \varphi \left(\frac{1}{2} \right) D^{-3/2} 2(z - \eta) = \frac{(z - \eta) \cos \varphi}{D^{3/2}}$$

Felhasználtuk, hogy: $\frac{\partial D}{\partial z} = 2(z - \eta).$

A **j** komponens:

$$-\frac{\partial}{\partial z} \left(\frac{\sin \varphi}{\sqrt{D}} \right) = -\sin \varphi \frac{\partial}{\partial z} \left(D^{-1/2} \right) = -\sin \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial z} = \sin \varphi \left(\frac{1}{2} \right) D^{-3/2} 2(z - \eta) = \frac{(z - \eta) \sin \varphi}{D^{3/2}}$$

Felhasználtuk, hogy: $\frac{\partial D}{\partial z} = 2(z - \eta)$

A **k** komponens első tagja:

$$\frac{\partial}{\partial x} \left(\frac{\cos \varphi}{\sqrt{D}} \right) = \cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial x} = -\frac{1}{2} \cos \varphi D^{-3/2} 2(x - r \cos \varphi) = -\frac{(x - r \cos \varphi) \cos \varphi}{D^{3/2}}$$

Felhasználtuk, hogy $\frac{\partial D}{\partial x} = 2(x - r \cos \varphi)$

A **k** komponens második tagja:

$$\frac{\partial}{\partial y} \left(\frac{\sin \varphi}{\sqrt{D}} \right) = \sin \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial y} = -\frac{1}{2} \sin \varphi D^{-3/2} 2(y - r \sin \varphi) = -\frac{(y - r \sin \varphi) \sin \varphi}{D^{3/2}}$$

Felhasználtuk, hogy $\frac{\partial D}{\partial y} = 2(y - r \sin \varphi)$

A **k** komponens két tagja összevonva:

$$\frac{\partial}{\partial x} \left(\frac{\cos \varphi}{\sqrt{D}} \right) + \frac{\partial}{\partial y} \left(\frac{\sin \varphi}{\sqrt{D}} \right) = \frac{1}{D^{3/2}} (-x \cos \varphi + r \cos^2 \varphi - y \sin \varphi + r \sin^2 \varphi) = \frac{r - x \cos \varphi - y \sin \varphi}{D^{3/2}}$$

$$\mathbf{B}(x, y, z) = \frac{A_{\max}}{2\pi} \int_{-l/2}^{l/2} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{(z - \eta) \cos \varphi}{D^{3/2}} + \mathbf{j} \frac{(z - \eta) \sin \varphi}{D^{3/2}} + \mathbf{k} \frac{r - x \cos \varphi - y \sin \varphi}{D^{3/2}} \right) d\varphi \right] d\eta$$

$$A_{\max} = \mu_0 \frac{NI}{l} \frac{r}{2}$$

$$\mathbf{B}(x, y, z) = \frac{\mu_0}{4\pi} \frac{NI}{l} r \int_{-l/2}^{l/2} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{(z - \eta) \cos \varphi}{D^{3/2}} + \mathbf{j} \frac{(z - \eta) \sin \varphi}{D^{3/2}} + \mathbf{k} \frac{r - x \cos \varphi - y \sin \varphi}{D^{3/2}} \right) d\varphi \right] d\eta$$

$$B_0 = \mu_0 \frac{NI}{l}$$

$$\mathbf{B}(x, y, z) = \frac{B_0}{4\pi} r \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{(z-\eta)\cos\varphi}{D^{3/2}} + \mathbf{j} \frac{(z-\eta)\sin\varphi}{D^{3/2}} + \mathbf{k} \frac{r-x\cos\varphi-y\sin\varphi}{D^{3/2}} \right) d\varphi \right] d\eta$$

$$\frac{\mathbf{B}(x, y, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{(z-\eta)\cos\varphi}{D^{3/2}} + \mathbf{j} \frac{(z-\eta)\sin\varphi}{D^{3/2}} + \mathbf{k} \frac{r-x\cos\varphi-y\sin\varphi}{D^{3/2}} \right) d\varphi \right] d\eta$$

$$\frac{B_x(x, y, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{(z-\eta)\cos\varphi}{D^{3/2}} d\varphi \right] d\eta$$

$$\frac{B_y(x, y, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{(z-\eta)\sin\varphi}{D^{3/2}} d\varphi \right] d\eta$$

$$\frac{B_z(x, y, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{r-x\cos\varphi-y\sin\varphi}{D^{3/2}} d\varphi \right] d\eta$$

$$D = x^2 + y^2 + (z-\eta)^2 + r^2 - 2r(x\cos\varphi + y\sin\varphi)$$

Forgásszimmetria miatt x, z sík metszetet vizsgálunk. Itt $y = 0$

$$D = x^2 + (z-\eta)^2 + r^2 - 2rx\cos\varphi \quad \text{Ez páros fv } \varphi\text{-ben.}$$

$$\frac{B_x(x, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{(z-\eta)\cos\varphi}{D^{3/2}} d\varphi \right] d\eta \quad \text{Az integrandus páros fv } \varphi\text{-ben tehát } B_x(x, z) \neq 0$$

$$\frac{B_y(x, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{(z-\eta)\sin\varphi}{D^{3/2}} d\varphi \right] d\eta \quad \text{Az integrandus páratlan fv } \varphi\text{-ben } B_y(x, z) = 0$$

$$\frac{B_z(x, z)}{B_0} = \frac{r}{4\pi} \int_{-1/2}^{1/2} \left[\int_{-\pi}^{\pi} \frac{r-x\cos\varphi}{D^{3/2}} d\varphi \right] d\eta \quad \text{Az integrandus páros fv } \varphi\text{-ben tehát } B_z(x, z) \neq 0$$

A nem nulla komponenseket az alábbiakban összefoglaljuk azzal a kiegészítéssel, hogy a nullával osztás megelőzése okán bevezetjük a δ huzalvastagságot.

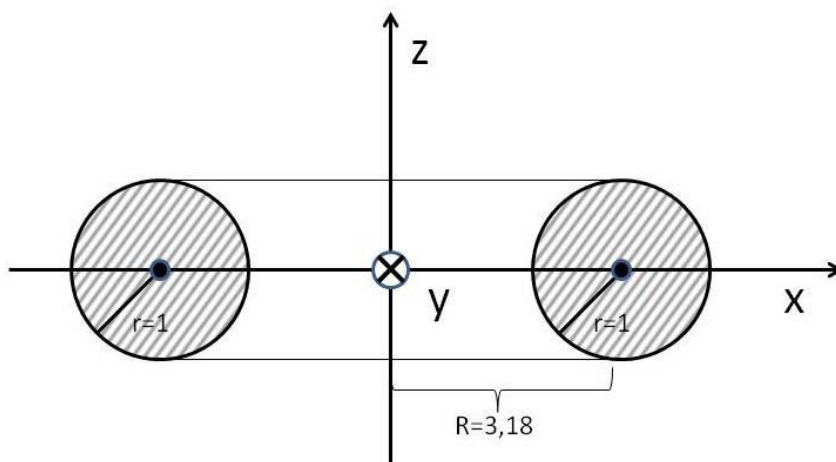
$$\frac{A_y(x, z)}{A_{\max}} = \frac{1}{2\pi} \int_{-1/2}^{1/2} \left(\int_{-\pi}^{\pi} \frac{\cos\varphi}{\sqrt{D+\delta}} d\varphi \right) d\eta$$

$$\frac{B_x(x, z)}{B_0} = \frac{r}{4\pi} \int_{-\pi/2}^{\pi/2} \left[\int_{-\pi}^{\pi} \frac{(z - \eta) \cos \varphi}{(\sqrt{D} + \delta)^3} d\varphi \right] d\eta$$

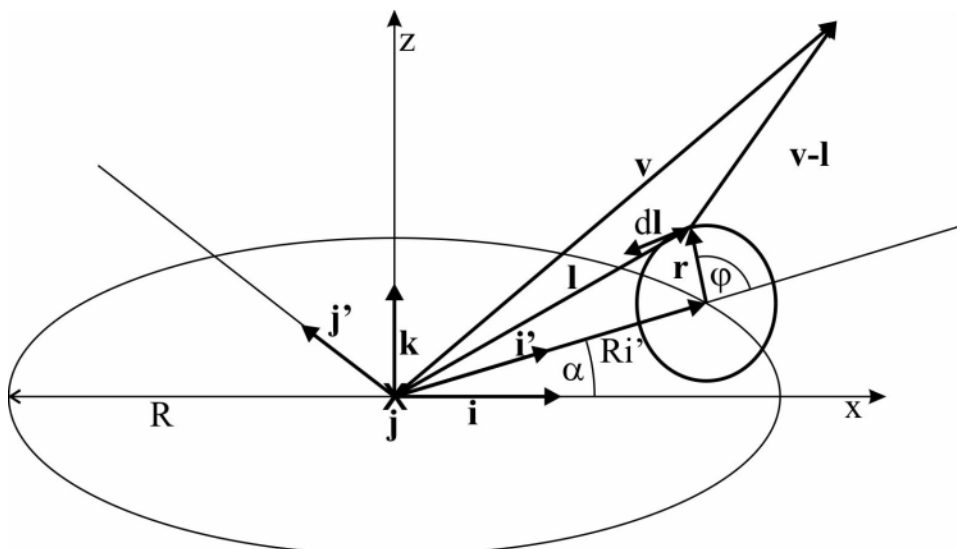
$$\frac{B_z(x, z)}{B_0} = \frac{r}{4\pi} \int_{-\pi/2}^{\pi/2} \left[\int_{-\pi}^{\pi} \frac{r - x \cos \varphi}{(\sqrt{D} + \delta)^3} d\varphi \right] d\eta$$

A numerikus számításoknál a következő paramétereket alkalmaztuk:
 A szolenoid rádiusza az egység $r = 1$, hossza $l = 20$, huzalvastagság $\delta = 10^{-2}$.

A toroid mágneses vektorpotenciál (**A**) terének és mágneses indukció (**B**) terének számítása:



A toroid metszete. Forgástengelye a z tengely, míg x és y a forgástengelyre merőleges síkot képez



$$\mathbf{A}^*(\mathbf{v}) = \frac{\mu_0}{4\pi} \int \frac{I^* d\mathbf{l}}{|\mathbf{v} - \mathbf{l}|}$$

$$\mathbf{i}' = \mathbf{i} \cos \alpha + \mathbf{j} \sin \alpha$$

$$\mathbf{j}' = \mathbf{i} \cos(90^\circ + \alpha) + \mathbf{j} \sin(90^\circ + \alpha) = \mathbf{i}(-\sin \alpha) + \mathbf{j} \cos \alpha$$

$$\mathbf{r} = \mathbf{i}' r \cos \varphi + \mathbf{k} r \sin \varphi = r \cos \varphi (\mathbf{i} \cos \alpha + \mathbf{j} \sin \alpha) + \mathbf{k} r \sin \varphi = \mathbf{i} r \cos \varphi \cos \alpha + \mathbf{j} r \cos \varphi \sin \alpha + \mathbf{k} r \sin \varphi$$

$$\mathbf{l} = R\mathbf{i}' + \mathbf{r} = R\mathbf{i} \cos \alpha + R\mathbf{j} \sin \alpha + \mathbf{i} r \cos \varphi \cos \alpha + \mathbf{j} r \cos \varphi \sin \alpha + \mathbf{k} r \sin \varphi$$

$$\mathbf{l} = \mathbf{i}(R \cos \alpha + r \cos \varphi \cos \alpha) + \mathbf{j}(R \sin \alpha + r \cos \varphi \sin \alpha) + \mathbf{k} r \sin \varphi$$

$$\frac{d\mathbf{l}}{d\varphi} = \mathbf{i}(-r \sin \varphi \cos \alpha) + \mathbf{j}(-r \sin \varphi \sin \alpha) + \mathbf{k} r \cos \varphi$$

$$d\mathbf{l} = [\mathbf{i}(-r \sin \varphi \cos \alpha) + \mathbf{j}(-r \sin \varphi \sin \alpha) + \mathbf{k} r \cos \varphi] d\varphi$$

$$\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{v} - \mathbf{l} = (x - R \cos \alpha - r \cos \varphi \cos \alpha)\mathbf{i} + (y - R \sin \alpha - r \cos \varphi \sin \alpha)\mathbf{j} + (z - r \sin \varphi)\mathbf{k}$$

$$|\mathbf{v} - \mathbf{l}|^2 = D = (x - R \cos \alpha - r \cos \varphi \cos \alpha)^2 + (y - R \sin \alpha - r \cos \varphi \sin \alpha)^2 + (z - r \sin \varphi)^2$$

A pitagoraszi összefüggés többszöri felhasználásával egyszerűbb alakra hozva:

$$D = x^2 + y^2 + z^2 + R^2 + r^2 - 2(Rx \cos \alpha + rx \cos \varphi \cos \alpha + Ry \sin \alpha + ry \cos \varphi \sin \alpha + rz \sin \varphi - Rr \cos \varphi)$$

$$\mathbf{A}^*(x, y, z) = \frac{\mu_0}{4\pi} \int \frac{I^* d\mathbf{l}}{\sqrt{D}}$$

$$I^* = \frac{NI}{2\pi} d\alpha$$

$$d\mathbf{A}(x, y, z) = \frac{\mu_0}{4\pi} \frac{NI}{2\pi} d\alpha \left(\mathbf{i} \int_{-\pi}^{\pi} \frac{-r \sin \varphi \cos \alpha}{\sqrt{D}} d\varphi + \mathbf{j} \int_{-\pi}^{\pi} \frac{-r \sin \varphi \sin \alpha}{\sqrt{D}} d\varphi + \mathbf{k} \int_{-\pi}^{\pi} \frac{r \cos \varphi}{\sqrt{D}} d\varphi \right)$$

$$A_0 = \mu_0 \frac{NI}{2R\pi} \frac{r}{2}$$

$$\frac{d\mathbf{A}(x, y, z)}{A_0} = \frac{R}{2r\pi} \left(\mathbf{i} \int_{-\pi}^{\pi} \frac{-r \sin \varphi \cos \alpha}{\sqrt{D}} d\varphi + \mathbf{j} \int_{-\pi}^{\pi} \frac{-r \sin \varphi \sin \alpha}{\sqrt{D}} d\varphi + \mathbf{k} \int_{-\pi}^{\pi} \frac{r \cos \varphi}{\sqrt{D}} d\varphi \right) d\alpha$$

$$\frac{\mathbf{A}(x, y, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\mathbf{i} \int_{-\pi}^{\pi} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} d\varphi + \mathbf{j} \int_{-\pi}^{\pi} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} d\varphi + \mathbf{k} \int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\alpha$$

Komponensek szerint:

$$\frac{A_x(x, y, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} d\varphi \right) d\alpha$$

$$\frac{A_y(x, y, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} d\varphi \right) d\alpha$$

$$\frac{A_z(x, y, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\alpha$$

Forgásszimmetria miatt x, z sík metszetet vizsgálunk. Itt $y = 0$

$$D = x^2 + z^2 + R^2 + r^2 - 2(Rx \cos \alpha + rx \cos \varphi \cos \alpha + rz \sin \varphi - Rr \cos \varphi) \quad \alpha\text{-ban páros fv}$$

$$\frac{A_x(x, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} d\varphi \right) d\alpha \quad \text{Az integrandus } \alpha\text{-ban páros fv tehát } A_y(x, z) \neq 0$$

$$\frac{A_y(x, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} d\varphi \right) d\alpha \quad \text{Az integrandus } \alpha\text{-ban páratlan fv } A_x(x, z) = 0$$

$$\frac{A_z(x, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D}} d\varphi \right) d\alpha \quad \text{Az integrandus } \alpha\text{-ban páros fv tehát } A_y(x, z) \neq 0$$

Általánosságban tehát:

$$\mathbf{A}(x, y, z) = A_0 \frac{R}{2\pi} \int_{-\pi}^{\pi} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} + \mathbf{j} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} + \mathbf{k} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \right] d\alpha$$

$$A_0 = \mu_0 \frac{NI}{2R\pi} \frac{r}{2}$$

$$\mathbf{A}(x, y, z) = \mu_0 \frac{NI}{2R\pi} \frac{r}{2} \frac{R}{2\pi} \int_{-\pi}^{\pi} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} + \mathbf{j} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} + \mathbf{k} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \right] d\alpha$$

$$B_o = \mu_0 \frac{NI}{2R\pi}$$

$$\mathbf{A}(x, y, z) = B_o \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left[\int_{-\pi}^{\pi} \left(\mathbf{i} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} + \mathbf{j} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} + \mathbf{k} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \right] d\alpha$$

A mágneses indukció vektorát a mágneses vektorpotenciál rotációjaként kapjuk

$$\mathbf{B} = \text{rot}\mathbf{A}$$

$$\mathbf{B}(x, y, z) = B_0 \frac{rR}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \text{rot} \left(\mathbf{i} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} + \mathbf{j} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} + \mathbf{k} \frac{\cos \varphi}{\sqrt{D}} \right) d\varphi \Big] d\alpha$$

Vizsgáljuk az integrandus vektort.

$$\text{rot} \left(\mathbf{i} \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} + \mathbf{j} \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} + \mathbf{k} \frac{\cos \varphi}{\sqrt{D}} \right) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-\sin \varphi \cos \alpha}{\sqrt{D}} & \frac{-\sin \varphi \sin \alpha}{\sqrt{D}} & \frac{\cos \varphi}{\sqrt{D}} \end{vmatrix} =$$

Az alábbiakban felhasználjuk a következő három összefüggést:

$$\frac{\partial D}{\partial x} = 2(x - R \cos \alpha - r \cos \varphi \cos \alpha)$$

$$\frac{\partial D}{\partial y} = 2(y - R \sin \alpha - r \cos \varphi \sin \alpha)$$

$$\frac{\partial D}{\partial z} = 2(z - r \sin \varphi)$$

Az \mathbf{i} komponens:

$$\frac{\partial}{\partial y} \left(\frac{\cos \varphi}{\sqrt{D}} \right) + \frac{\partial}{\partial z} \left(\frac{\sin \varphi \sin \alpha}{\sqrt{D}} \right)$$

Kifejtve:

$$\begin{aligned} & \cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial y} + \sin \varphi \sin \alpha \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial z} = \\ & = \cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} 2(y - R \sin \alpha - r \cos \varphi \sin \alpha) + \sin \varphi \sin \alpha \left(-\frac{1}{2} \right) D^{-3/2} 2(z - r \sin \varphi) = \\ & = -\frac{\cos \varphi (y - R \sin \alpha - r \cos \varphi \sin \alpha) + \sin \varphi \sin \alpha (z - r \sin \varphi)}{D^{3/2}} = \\ & = \frac{-y \cos \varphi + (R \cos \varphi + r - z \sin \varphi) \sin \alpha}{D^{3/2}} \end{aligned}$$

A **j** komponens:

$$-\frac{\partial}{\partial x} \left(\frac{\cos \varphi}{\sqrt{D}} \right) - \frac{\partial}{\partial z} \left(\frac{\sin \varphi \cos \alpha}{\sqrt{D}} \right) =$$

Kifejtve:

$$\begin{aligned} & -\cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial x} - \sin \varphi \cos \alpha \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial z} = \\ & = -\cos \varphi \left(-\frac{1}{2} \right) D^{-3/2} 2(x - R \cos \alpha - r \cos \varphi \cos \alpha) - \sin \varphi \cos \alpha \left(-\frac{1}{2} \right) D^{-3/2} 2(z - r \sin \varphi) = \\ & = \frac{\cos \varphi (x - R \cos \alpha - r \cos \varphi \cos \alpha) + \sin \varphi \cos \alpha (z - r \sin \varphi)}{D^{3/2}} = \\ & = \frac{x \cos \varphi - (R \cos \varphi + r - z \sin \varphi) \cos \alpha}{D^{3/2}} \end{aligned}$$

A **k** komponens:

$$\frac{\partial}{\partial x} \left(-\frac{\sin \varphi \sin \alpha}{\sqrt{D}} \right) + \frac{\partial}{\partial y} \left(\frac{\sin \varphi \cos \alpha}{\sqrt{D}} \right) =$$

Kifejtve:

$$\begin{aligned} & -\sin \varphi \sin \alpha \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial x} + \sin \varphi \cos \alpha \left(-\frac{1}{2} \right) D^{-3/2} \frac{\partial D}{\partial y} = \\ & = -\sin \varphi \sin \alpha \left(-\frac{1}{2} \right) D^{-3/2} 2(x - R \cos \alpha - r \cos \varphi \cos \alpha) + \sin \varphi \cos \alpha \left(-\frac{1}{2} \right) D^{-3/2} 2(y - R \sin \alpha - r \cos \varphi \sin \alpha) = \\ & = \frac{\sin \varphi \sin \alpha (x - R \cos \alpha - r \cos \varphi \cos \alpha) - \sin \varphi \cos \alpha (y - R \sin \alpha - r \cos \varphi \sin \alpha)}{D^{3/2}} = \\ & = \frac{(y \cos \alpha - x \sin \alpha) \sin \varphi}{D^{3/2}} \end{aligned}$$

Behelyettesítjük a komponenseket:

$$\frac{B_x(x, y, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-y \cos \varphi + (R \cos \varphi + r - z \sin \varphi) \sin \alpha}{D^{3/2}} d\varphi \right) d\alpha$$

$$\frac{B_y(x, y, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{x \cos \varphi - (R \cos \varphi + r - z \sin \varphi) \cos \alpha}{D^{3/2}} d\varphi \right) d\alpha$$

$$\frac{B_z(x, y, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{(y \cos \alpha - x \sin \alpha) \sin \varphi}{D^{3/2}} d\varphi \right) d\alpha$$

Forgásszimmetria miatt x, z sík metszetet vizsgálunk. Itt $y = 0$

$$D = x^2 + z^2 + R^2 + r^2 - 2(Rx \cos \alpha + rx \cos \varphi \cos \alpha + rz \sin \varphi - Rr \cos \varphi) \quad \alpha\text{-ban páros fv}$$

Az integrandus α -ban páratlan fv $B_x(x, z) = 0$

$$\frac{B_x(x, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{(R \cos \varphi + r - z \sin \varphi) \sin \alpha}{D^{3/2}} d\varphi \right) d\alpha$$

Az integrandus α -ban páros fv tehát $B_y(x, z) \neq 0$

$$\frac{B_y(x, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{x \cos \varphi - (R \cos \varphi + r - z \sin \varphi) \cos \alpha}{D^{3/2}} d\varphi \right) d\alpha$$

Az integrandus α -ban páratlan fv tehát $B_z(x, z) = 0$

$$\frac{B_z(x, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-x \sin \alpha \sin \varphi}{D^{3/2}} d\varphi \right) d\alpha$$

A nem nulla komponenseket az alábbiakban összefoglaljuk azzal a kiegészítéssel, hogy a nullával osztás megelőzése okán bevezetjük a δ huzalvastagságot.

$$\frac{A_x(x, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{-\sin \varphi \cos \alpha}{\sqrt{D} + \delta} d\varphi \right) d\alpha$$

$$\frac{A_z(x, z)}{A_0} = \frac{R}{2\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{\cos \varphi}{\sqrt{D} + \delta} d\varphi \right) d\alpha$$

$$\frac{B_y(x, z)}{B_0} = \frac{rR}{4\pi} \int_{-\pi}^{\pi} \left(\int_{-\pi}^{\pi} \frac{x \cos \varphi - (R \cos \varphi + r - z \sin \varphi) \cos \alpha}{(\sqrt{D} + \delta)^3} d\varphi \right) d\alpha$$

A numerikus számításoknál a következő paramétereket alkalmaztuk:

A toroid meneteinek rádiusza az egység $r = 1$, a toroid alakzat rádiusza $R = 3.18$, (ez a R rádiusz éppen 20 kerületű tekercset eredményez, mintha az előző számítás szolenoid tekercsét kör alakúvá hajlítottuk volna) továbbá a huzalvastagság $\delta = 10^{-2}$.